Problem J06E.2

(a) Since $\mu_r \gg 1$, to a good approximation the $\mathbf{H}$ field is confined to within the ring. The reluctance of the ring is

$$\mathcal{R}_{\text{ring}} = \frac{2\pi R}{\mu_r \mu_0 \pi a^2},$$

and the reluctance of the gap is

$$\mathcal{R}_{\text{gap}} = \frac{2s}{\mu_0 \pi a^2}.$$

The magneto-motive force is $NI = 640\text{A}\,\text{turns}$, so the magnetic flux through the circuit is

$$\Phi_B = \frac{NI}{\mathcal{R}_{\text{tot}}} = \pi a^2 \mu_0 NI \left(\frac{2\pi R}{\mu_r} + 2s\right)^{-1} = 39\,\mu\text{Wb}.$$

Thus we have

$$|\mathbf{B}| = \frac{\Phi_B}{A} = \mu_0 NI \left(\frac{2\pi R}{\mu_r} + 2s\right)^{-1} = 58\,\text{mT}$$

everywhere throughout the circuit. Within the ring, we have

$$|\mathbf{H}| = \frac{|\mathbf{B}|}{\mu_0 \mu_r} = 115\,\text{A/m},$$

and within the gap, we have

$$|\mathbf{H}| = \frac{|\mathbf{B}|}{\mu_0} = 46\,\text{kA/m}.$$

(b) The total magnetic energy is

$$\begin{aligned} U &= \frac{1}{2}\int \mathbf{B}\cdot\mathbf{H}\,\mathrm{d}V \\ &= \frac{\pi a^2 \mu_0}{2} N^2 I^2 \left(\frac{2\pi R}{\mu_r} + 2s\right)^{-1} \\ &= 13\,\text{mJ}. \end{aligned}$$

(c) The coil's self-inductance L satisfies

$$U = \frac{1}{2} L I^2,$$

yielding

$$L = \pi a^2 \mu_0 N^2 \left(\frac{2\pi R}{\mu_r} + 2s\right)^{-1} = 41\,\text{mH}.$$

As expected, we have $N\Phi_B = LI$.

(d) The total force on the coil is

$$F = \frac{\partial U}{\partial s} = \pi a^2 \mu_0 N^2 I^2 \left(\frac{2\pi R}{\mu_r} + 2s\right)^{-2} = 70\,\text{nN},$$

corresponding to a mass

$$m = \frac{F}{g} = \boxed{7.1\,\mu\text{g}}.$$

Time: 25 m 16 s