1 January 2005, Mechanics, Problem 1

At each point of contact of the rope with the cylinder, we can write the condition for no slipping as:

$$F_{left} = F_{right} + F_{friction} = F_{right} + \mu_s N$$

where the force F is applied "all the way on the left" and the force f is applied "all the way on the right" (in fact, the rope is curved around the cylinder, "left" and "right" are hard to define, but I just mean to define a direction within the rope, call it θ if you like). The normal force is applied exactly at the point of contact, of width $\delta\theta$, so in fact we can say it's a "δN". The value of δN is given by the change of direction of the tension. If we are looking at a very small angle $\delta\theta$, then we can write:

$$\delta N = F_r sin\left(\frac{\delta\theta}{2}\right) + F_l sin\left(\frac{\delta\theta}{2}\right) \approx \frac{\delta\theta}{2}(F_r + F_l)$$

Assuming that $\delta\theta$ is very small, and solving for F_r we get:

$$F_r = F_l(1 - \mu_s\delta\theta)$$

The rope is in contact with the cylinder for a distance l, which is an angle of $\frac{l}{2\pi r}2\pi$, and in terms of $\delta\theta$ is $\frac{l}{r\delta\theta}\delta\theta$. To calculate the force at a point $\delta\theta$ away from the beginning of the rope, you have to calculate F_r by plugging in $F_l = F$ and then reiterating the procedure once. To calculate what happens after an angle of $\frac{l}{r\delta\theta}\delta\theta$ you have to reiterate $\frac{l}{r\delta\theta}$ times:

$$f = lim_{\delta\theta\to 0}(1 - \mu_s\delta\theta)^{\frac{l}{r\delta\theta}}$$

The limit can be evaluated using one of l'Hospital's theorems:

$$lim_{x\to 0}(1 + x)^{1/x} = e$$

$$f = Fe^{-\mu_s l/r} \tag{1}$$

To make it possible for a kid to hold an ocean liner, you have to choose a cylinder that is thin and with a high coefficient of friction, an wrap the rope many times around the cylinder.