1 January 2005, Electromagnetism, Problem 3

IMPORTANT: I spoke to graduate student Diego Hofman, who had this problem on his prelim. He made me notice that the profile is incorrectly written in the problem, and that is why we cannot get sensible answers. Therefore, this problem should not be taken very seriously.

1.1 (a)

Since the field is uniform, the dipole moment is constant.

$$\boldsymbol{F} = \nabla(\boldsymbol{Ep}) = 0 \tag{1}$$

1.2 (b)

The field is nonzero only inside the beam. Since $I = E^2$, we can write:

$$\boldsymbol{F} = \nabla(-\alpha a^3 I)$$
$$I = I(1 - \Theta(\rho - w_0))$$

where Θ is a step function. Thus,

$$\boldsymbol{F} = \alpha a^3 I \delta(\rho - w_0)\hat{\rho} \tag{2}$$

1.3 (c)

Applying the same method as above, we get (inside the beam):

$$\boldsymbol{F} = \frac{\alpha a^3 P}{\pi w_0^2} \frac{1}{\left(1 + \left(\frac{z}{w_0}\right)^2\right)^2} \frac{2z}{w_0^2}\hat{z}$$

This force is 0 when $z = 0$, and hence that is the equilibrium (stable) point.