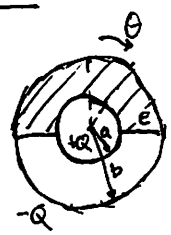
JOSE.1



Denote region with permitivity ϵ by sub-index u and vacuum with sub-index L.

a.) From $\nabla^2\phi = 0$ in region of $a<r<b$ we get azimuthally symmetric solution

$$\phi(r,\theta) = \sum_n \left(A_n r^n + \frac{B_n}{r^{n+1}}\right) P_n(\cos\theta)$$

then using BC of equipotential surface at $r=a$ and $r=b$ we get

$$\phi_u(r,\theta) = A_0 + \frac{B_0}{r}$$

$$\phi_L(r,\theta) = C_0 + \frac{D_0}{r}$$

then from continuity of tangential $\vec{E}$ on interface:

$$\vec{E}_u \cdot \hat{r} = \vec{E}_L \cdot \hat{r}$$

$$\frac{B_0}{r^2} = \frac{D_0}{r^2}$$

$$B_0 = D_0$$

thus we know that $\vec{E} = \frac{B_0}{r^2}\hat{r}$ for $a<r<b$ then

$$\int \vec{D} \cdot d\vec{A} = Q \quad \cdots \text{ for } a<r<b$$

$$\epsilon \frac{B_0}{r^2} \cdot 2\pi r^2 + \epsilon_0 \frac{B_0}{r^2} 2\pi r^2 = Q$$

$$B_0 = \frac{Q}{2\pi(\epsilon_0 + \epsilon)}$$

Therefore we have

$$\vec{E} = \begin{cases} \dfrac{Q}{2\pi(\epsilon+\epsilon_0)r^2}\hat{r} & \text{for } a<r<b \\ 0 & \text{otherwise} \end{cases}$$

$$\vec{D} = \begin{cases} \dfrac{\epsilon Q}{2\pi(\epsilon+\epsilon_0)r^2}\hat{r} & \text{for } a<r<b,\ 0 \leq \theta < \frac{\pi}{2} \\ \dfrac{\epsilon_0 Q}{2\pi(\epsilon+\epsilon_0)r^2}\hat{r} & \text{for } a<r<b,\ \frac{\pi}{2} < \theta \leq \pi \\ 0 & \text{otherwise} \end{cases}$$

b.) In the dielectric for $a<r<b$ and $0\leqslant\theta<\frac{\pi}{2}$ we have

$$\vec{P} = \vec{D} - \epsilon_0 \vec{E}$$
$$= \frac{(\epsilon-\epsilon_0)Q}{2\pi(\epsilon+\epsilon_0)r^2}\hat{r}$$

thus the bound charge densities are:

- $\nabla\cdot\vec{P} = -\rho_b$

  $\rho_b = 0$ //

- At $r=a$

  $$\sigma_a = -\vec{P}\cdot\hat{r}\Big|_{r=a}$$
  $$= -\frac{(\epsilon-\epsilon_0)Q}{2\pi(\epsilon+\epsilon_0)a^2}$$ //

- At $r=b$

  $$\sigma_b = \vec{P}\cdot\hat{r}\Big|_{r=b}$$
  $$= \frac{(\epsilon-\epsilon_0)Q}{2\pi(\epsilon+\epsilon_0)b^2}$$

- On interface

  $$\sigma_{\pm i} = \pm\vec{P}\cdot\hat{\theta}$$
  $$= 0$$ //