January 2004 EM # 3

a. 

$$\vec{x}(t) = a\cos(wt)\hat{x} + a\sin(wt)\hat{y}$$

$$\ddot{\vec{x}}(t) = -aw^2[\cos(wt)\hat{x} + \sin(wt)\hat{y}]$$

$$\ddot{\vec{d}}(t) = q\ddot{\vec{x}}(t) = -qaw^2[\cos(wt)\hat{x} + \sin(wt)\hat{y}]$$

$$\ddot{\vec{d}}(t) \times \hat{n} = -qaw^2[\cos(wt)\hat{x} + \sin(wt)\hat{y}] \times (\sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z})$$

$$= -qaw^2[\sin\theta\sin\phi\cos(wt)\hat{z} - \cos\theta\cos(wt)\hat{y} - \sin\theta\cos\phi\sin(wt)\hat{z} + \cos\theta\sin(wt)\hat{x}]$$

$$\ddot{\vec{d}}(t) \times \hat{n} = -qaw^2[-\sin\theta\sin(wt-\phi)\hat{z} + \cos\theta(\sin(wt)\hat{x} + \cos(wt)\hat{y})]$$

$$|\ddot{\vec{d}}(t) \times \hat{n}|^2 = q^2a^2w^4[\sin^2\theta\sin^2(wt-\phi) + \cos^2\theta]$$

$$\frac{dP}{d\Omega} = \frac{1}{4\pi c^3}|\ddot{\vec{d}}(t) \times \hat{n}|^2$$

$$\frac{dP}{d\Omega} = \frac{1}{4\pi c^3}q^2a^2w^4[\sin^2\theta\sin^2(wt-\phi) + \cos^2\theta]$$

$$\frac{dP}{d\Omega} = \frac{q^2a^2w^4}{4\pi c^3}[1 - \sin^2\theta\cos^2(wt-\phi)]$$

$$\left\langle\frac{dP}{d\Omega}\right\rangle = \frac{q^2a^2w^4}{4\pi c^3}\left(1 - \frac{1}{2}\sin^2\theta\right) \qquad v = wa \Rightarrow w = \frac{v}{a}$$

$$\left\langle\frac{dP}{d\Omega}\right\rangle = \frac{q^2v^4}{4\pi a^2c^3}\left(1 - \frac{1}{2}\sin^2\theta\right)$$

b. The radiation is emitted at the same frequency as the origional EM wave and the orbit of the particle

$$w = \frac{|\vec{v}|}{a}$$

c. $P = \int\left\langle\frac{dP}{d\Omega}\right\rangle d\Omega$

$$P = \frac{q^2v^4}{4\pi a^2c^3}\int_0^\pi\left(1 - \frac{1}{2}\sin^2\theta\right)\sin\theta\, d\theta$$

$$= \frac{q^2v^4}{4\pi a^2c^3}\int_0^\pi\left(\frac{1}{2} + \frac{1}{2}\cos^2\theta\right)d(-\cos\theta)$$

$$= \frac{q^2v^4}{4\pi a^2c^3}\cdot -\frac{1}{2}\left(\cos\theta + \frac{\cos^3\theta}{3}\Big|_0^\pi\right)$$

$$= \frac{q^2v^4}{8\pi a^2c^3}\left(\frac{8}{3}\right)$$

$$P = \frac{q^2v^4}{3\pi a^2c^3}$$

$$m\ddot{\vec{x}}(t) = q\vec{E}(t)$$

$$\vec{E}(t) = \frac{m}{q}\ddot{\vec{x}}(t) = -\frac{maw^2}{q}[\cos(wt)\hat{x} + \sin(wt)\hat{y}]$$

$$\vec{S} = \frac{c}{4\pi}\vec{E} \times \vec{B}$$

$$\vec{S} = \frac{c}{4\pi}\left(\frac{maw^2}{q}\right)^2[\cos(kz-wt)\hat{x} + \sin(kz-wt)\hat{y}] \times [-\sin(kz-wt)\hat{x} + \cos(kz-wt)\hat{y}]$$

$$|\vec{S}| = \frac{c}{4\pi}\left(\frac{maw^2}{q}\right)^2 = \frac{c}{4\pi}\left(\frac{mv^2}{qa}\right)^2$$

$$\sigma = \frac{P}{|\vec{S}|} = \frac{1}{3\pi c^3}\left(\frac{qv^2}{a}\right)^2 \cdot \frac{4\pi}{c}\left(\frac{qa}{mv^2}\right)^2$$

$$\sigma = \frac{4}{3c^4}\left(\frac{q^2}{m}\right)^2$$

$$\sigma = \frac{1}{3}\left(\frac{2q^2}{mc^2}\right)^2$$