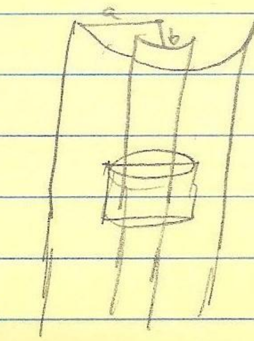
J04-1

Capacitors

a) give the inner cylider charge per length λ



we know the electric field is radial, so take a concentric cylinder of length h and radius r

$$\int \vec{E}\cdot d\vec{a} = 4\pi Q_{enc}$$

$$2\pi r \cancel{h}|E| = 4\pi \lambda \cancel{h}$$

$$|E| = \frac{2\lambda}{r}$$

$$\vec{E} = \frac{2\lambda}{r}\hat{r}$$

now $\vec{E} = -\nabla V$ ~~&~~

so $|E| = -\frac{dV}{dr}$

want $U = -\int_b^a |E|\, dr$

$$U = -2\lambda\int_b^a \frac{dr}{r} = -2\lambda\ln\left(\frac{a}{b}\right) = -2\frac{Q}{\ell}\ln\left(\frac{a}{b}\right)$$

so $\boxed{|Q| = \frac{U\ell}{2\ln\left(\frac{a}{b}\right)}}$ $\qquad Q = CV$

$$\vec{E}(\rho) = \frac{U}{\rho\ln\left(\frac{a}{b}\right)}\hat{\rho}$$

b) well, Faraday's Law $\vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{d\vec{B}}{dt}$

$$\int \vec{E}\cdot d\vec{l} = -\frac{1}{c}\frac{\partial\Phi}{dt}$$

$$|E|(2\pi r) = -\frac{1}{c}(\pi r^2)\frac{dB}{dt}$$

$$|E| = -\frac{r}{2c}\frac{dB}{dt}$$

$$\vec{F} = -\frac{qr}{2c}\frac{dB}{dt}\hat{\theta}$$

$$\tau = \vec{r}\times\vec{F} = -\frac{qr^2}{2c}\frac{dB}{dt}\hat{z}$$

so $\frac{d\vec{L}}{dt} = -\frac{qr^2}{2c}\frac{dB}{dt}\hat{z}$

$$\int_0^L d\vec{L} = -\int_{B_0}^{0}\frac{qr^2}{2c}\,d\vec{B}$$

$$L = \frac{qr^2}{2c}B_0$$

assume outer is negative Q

the $L_i = \frac{Qb^2}{2c}B_0 = \boxed{\frac{U l b^2 B_0}{4c\ln(\frac{a}{b})}}$

$$\boxed{L_o = -\frac{U l a^2 B_0}{4c\ln(\frac{a}{b})}}$$

c) well $\frac{c}{4\pi}\vec{E}\times\vec{B} = \frac{c}{4\pi}\left(\frac{U}{\rho\ln(\frac{a}{b})}\right)(B_0)(\hat{\rho}\times\hat{z}) = -\frac{cU B_0}{4\pi\rho\ln(\frac{a}{b})}\hat{\theta}$

now $\vec{L} = \int_{\text{vol}}\vec{p}\times\vec{S}\,d^3r = -\frac{cUB_0 l}{2\ln(\frac{a}{b})}\cdot\int_b^a r\,dr = -\frac{cUB_0 l}{4\ln(\frac{a}{b})}(a^2-b^2)$

$$\begin{vmatrix} \hat{\rho} & \hat{\theta} & \hat{z} \\ \rho & z & z \\ & S & \end{vmatrix} = \hat{r}(-Sz) + \hat{z}(\rho S)$$

d) ~~increase~~ decreases