Seth Dorfman Prelims January 2, 2006

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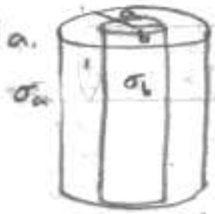
1) a.



$\oint \vec{E}\cdot d\vec{a} = 4\pi Q_{enc}$

$E \cdot 2\pi \rho \ell = 4\pi \sigma_b 2\pi b \,\ell$

$\vec{E} = \frac{4\pi \sigma_b b}{\rho}\,\hat{\rho}$

(Define $U = V_b - V_a$)

$V = -\int \vec{E}\cdot d\vec{\ell}$

$= -\int_a^b \frac{4\pi \sigma_b b}{\rho}\, d\rho$

$U = 4\pi \sigma_b b \ln\left(\frac{a}{b}\right)$

$\sigma_b = \frac{U}{4\pi b \ln(a/b)}$ $\left(\begin{matrix}\sigma_b 2\pi b \\ = -\sigma_a 2\pi a\end{matrix}\right)$

$\sigma_a = \frac{-U}{4\pi a \ln(a/b)}$

$\vec{E} = \frac{U}{\rho \ln(a/b)}\,\hat{\rho}$

b. $\vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$

$\frac{d\vec{L}}{dt} = \vec{\rho}\times\vec{F} = (\vec{\rho}\times\sigma\vec{E})\cdot \text{Area}^{\text{Surface}}$

$\oint \vec{E}\cdot d\vec{\ell} = -\frac{1}{c}\int \frac{\partial \vec{B}}{\partial t}\cdot d\vec{A}$

For the inner cyclinder:

$E \cdot 2\pi b = -\frac{1}{c}\frac{\partial B}{\partial t}\pi b^2$

$\frac{d\vec{L}}{dt} = -b\sigma_b E\,\hat{z}\cdot 2\pi b \ell = +b\sigma_b \ell\left(-\frac{1}{c}\frac{\partial \vec{B}}{\partial t}\pi b^2\right)$

$\frac{d\vec{L}}{dt} = +\frac{\pi b^3 \ell \sigma_b}{c}\frac{\partial \vec{B}}{\partial t}$

$\Rightarrow \vec{L}_i = +\frac{\pi b^3 \ell \sigma_b}{c} B_o\,\hat{z} = \frac{+Ub^2 \ell B_o}{4c\ln(a/b)}\,\hat{z}$

For the outer cylinder:

$E \cdot 2\pi a = -\frac{1}{c}\frac{\partial B}{\partial t}\pi a^2$

$\frac{d\vec{L}}{dt} = -a\sigma_a E\,\hat{z}\; 2\pi a \ell = a\sigma \ell\left(-\frac{1}{c}\frac{\partial \vec{B}}{\partial t}\pi a^2\right)$

$\frac{d\vec{L}}{dt} = +\frac{\pi a^3 \ell \sigma_a}{c}\frac{\partial \vec{B}}{\partial t}$

$\Rightarrow \vec{L}_o = \frac{\pi a^3 \ell \sigma_a}{c} B_o\,\hat{z} = \frac{-Ua^2 \ell B_o}{4c\ln(a/b)}\,\hat{z}$

c. $\vec{S} = \frac{1}{4\pi c}\vec{E}\times\vec{B}$

$= \frac{1}{4\pi c}\left(\frac{U}{\rho \ln(a/b)}\hat{\rho}\right)\times(B_o\hat{z})$

$\vec{S} = \frac{-UB_o}{4\pi c\,\rho \ln(a/b)}\,\hat{\phi}$

$\vec{j} = \vec{\rho}\times\vec{S}$

$= \vec{\rho}\times\left(\frac{-UB_o}{4\pi c\,\rho\ln(a/b)}\hat{\phi}\right)$

$\vec{j} = \frac{-UB_o}{4\pi c\ln(a/b)}\,\hat{z}$

$\vec{L}_{field} = \frac{-UB_o}{4\pi c\ln(a/b)}\cdot(\pi a^2 - \pi b^2)\ell$

$\vec{L}_{field} = \frac{UB_o \ell}{4c\ln(a/b)}(b^2 - a^2)\,\hat{z}$

$\vec{L}_{field} = \vec{L}_i + \vec{L}_o$ ✓

d. The rotating cylinders produce field in the $\hat{z}$ direction. Thus the above calculation overestimates the change in $\vec{B}$ and the magnitude of the anglular momentum would decrease if this were taken into account.