

We use a gaussian integral around the inner cylinder:

$$\int E da = 4\pi \int \rho dv \Rightarrow 2\pi r E = 4\pi \sigma_i 2\pi b \Rightarrow E = \frac{4\pi \sigma_i b}{r}$$

The potential difference is given by:

$$U = \int_b^a E dr = 4\pi \sigma_i b \ln a - 4\pi \sigma_i b \ln b$$

Solving for  $\sigma_i$ :

$$\sigma_i = \frac{U}{4\pi b \ln(a/b)}$$

The potential is given by:

$$\Phi = 4\pi \sigma_i b \ln r$$

So that the charge on the outer cylinder is given by:

$$\sigma_o = -\frac{1}{4\pi} \nabla \Phi \cdot \hat{n} = -\frac{1}{4\pi} \left( \frac{\partial \Phi}{\partial r} \right)_{r=a} = -\frac{\sigma_i b}{a}$$

We can integrate the charge densities to get the total charge:

$$Q_i = 2\pi b l \sigma_i = \frac{U}{2 \ln(a/b)}$$

$$Q_o = 2\pi a l \sigma_o = -l \frac{U}{2 \ln(a/b)}$$

And the electric field is given by:

$$\vec{E} = \frac{U}{\ln(a/b)} \frac{\hat{r}}{r}$$

The Faraday's Law gives:

$$2\pi b \mathbf{E} = -\frac{1}{c} \frac{d}{dt} \int_S \mathbf{B} \cdot \mathbf{n} da$$

So that for the inner cylinder:

$$\mathcal{E}_i = \frac{1}{c} \frac{B_0 b}{t_0} \frac{b}{2}$$

The torque will be:

$$\dot{L} = r \mathbf{E} Q \hat{z}$$

So that the induced angular momentum will be:

$$L_i = b \mathbf{E}_i Q_i t_0 \hat{z} = \frac{1}{c} B_0 \frac{b^2}{2} l \frac{U \hat{z}}{2 \ln(a/b)} = \frac{b^2 l B_0 U \hat{z}}{4c \ln(a/b)}$$

For the outer cylinder:

$$\mathcal{E}_o = \frac{1}{c} \frac{B_0 a}{t_0} \frac{a}{2}$$

So that:

$$L_o = a \mathbf{E}_o Q_o t_0 \hat{z} = -\frac{1}{c} B_0 \frac{a^2}{2} l \frac{U \hat{z}}{2 \ln(a/b)} = -\frac{a^2 l B_0 U \hat{z}}{4c \ln(a/b)}$$

And the total angular momentum is:

$$L = L_i + L_o = \frac{l B_0 U}{4c \ln(a/b)} \hat{z} (b^2 - a^2)$$

The field momentum is given by the **Poynting Vector**:

$$\mathbf{S} = \vec{\mathbf{E}} \times \vec{\mathbf{B}} / 4\pi c = \frac{U}{\ln(a/b)} \frac{1}{r} B_0 \frac{1}{4\pi c} \hat{\theta}$$

So that its angular momentum is:

$$L = \int \mathbf{r} \times \mathbf{S} dv = -2\pi l \hat{z} \int_b^a r \frac{U}{\ln(a/b)} \frac{1}{r} B_0 \frac{1}{4\pi c} r dr$$

where the integral over  $z$  and  $\theta$  is already done. Integrating over  $r$ :

$$L = \frac{l U B_0 \hat{z}}{4c \ln(a/b)} (b^2 - a^2)$$

The momentum would decrease, as the cylinders turn to oppose the change in  $\vec{\mathbf{B}}$  in the same direction as before. This field would have angular momentum in the same direction, so that the cylinders themselves would have less angular momentum.