

J03T.1

We can start by denoting the variables during the quasistatic compressing process. The pressure of gas on right and left side are denoted as P_R, P_L . The volume of both sides are denoted as V_R, V_L . The initial temperature is $T_0 = 305.15K$ and we denote the temperature increase during the process as δT .

For ideal gas we have:

$$P_R V_R = n_R R (T_0 + \delta T)$$

$$P_L V_L = n_L R (T_0 + \delta T)$$

At the very beginning, we have $P_R = P_L, V_R = 4V_L = 20L$, so we get $n_R = 4n_L = 4n$.

During the process, we have energy conservation equation,

$$- \int_{V_0}^{V_R} (P_R - P_L) dV_R = \frac{3}{2} (n_R + n_L) R \delta T + C_m M \delta T$$

In which, the specific heat and mass of water are denoted as C_m , and, M . Then use the first two equations to eliminate $P_L, \delta T$, we have,

$$- \int_{V_0}^{V_R} \left(P_R - \frac{P_R V_R}{4(V_t - V_R)} \right) = \left(\frac{15}{2} n R + C_m M \right) \left(\frac{P_R V_R}{4nR} - T_0 \right)$$

In which, V_t is the total volume which is 20L, we denote the new constant $t = \frac{\frac{15}{2} n R + C_m M}{4nR}$, by taking the derivative of the previous equation, we have:

$$-\frac{1}{t} \left(P_R - \frac{P_R V_R}{4(V_t - V_R)} \right) = \frac{dP_R}{dV_R} V_R + P_R$$

By integral the equation we get:

$$\left(1 + \frac{1}{t}\right) \ln V_R + \ln P_R + \frac{1}{4t} \ln(V_t - V_R) = \text{Const.}$$

Then we can use the final temperature increase $\delta T = 1K$ to get the final $V_R = 10.0328L$, then we have $t \simeq 127.08$, then we got the answer to the first question is :

$$n_L = 1.0048 \text{ mol}, n_R = 4.0194 \text{ mol}$$

For the seconde problem, instantly we can have the presure:

$$P_R = \frac{n_R R (T_0 + \delta T)}{V_R} = 5.0647 \text{ atm}$$

When the water is removed, the equation:

$$\left(1 + \frac{1}{t}\right) \ln V_R + \ln P_R + \frac{1}{4t} \ln(V_t - V_R) = \text{Const.}$$

is still valid, with only t changed into $t = \frac{\frac{15}{2}nR}{4nR} = \frac{15}{8}$,

Then we can solve the equation and get $V_R = 11.6876L$ after quasistatic compression.

Then we will be able to compute the T_2 after the second process,

$$T_2 = \frac{P_R V_R}{n_R R} = 357.82K$$

Then the pressure after the second process is,

$$P_R = \frac{n_R R T_2}{V_R} = 5.9195 \text{ atm}$$

For the third question, we are asked to calculate the entropy. Remember that the entropy of ideal gas is only a state function of T and V, with the form taken:

$$S = \frac{3}{2}nR \ln T + nR \ln V$$

for water we have:

$$\Delta S = C_m M \ln \frac{T + \delta T}{T}$$

After calculation, for the first process we have:

$$\Delta S = t \times 4nR \ln \frac{T_0 + \delta T}{T_0} = 13.8939 J/K$$

For the second process, we have:

$$\Delta S = t \times 4nR \ln \frac{T_2}{T_0 + \delta T} = 9.7717 J/K$$

For the first process, most of the heat is absorbed by the water. The water is like a heat bath or reservoir, in order to reach the critical pressure, one may need make the system more "chaotic" or increase more entropy. The second process, the gas heats itself more when compressing, and they don't need additional entropy to heat water. So the entropy increase is less in the second process.

Due to the random number I have got here, I strongly need a double check to my answer.

2 thoughts on "J03T.1"



M

December 12, 2013 at 11:39 pm

Your solution of parts (a) and (b) is correct and seems to be very good.

However, I must say that explanation is poor. I had to solve the problem myself and only then I was able to understand what was going on in your solution. In general, when you are writing up something (especially in science), you probably want it to be useful, which means that reading and understanding your solution for another person should take less time than just solving the problem.

Regarding the part (c), I'm satisfied both with your solution and explanation. Probably, it would make more sense to compare the entropies if the second process started from the same temperature T_0 , because it would clearly demonstrate the effect of the presence of water on the entropy change. But it's still OK.

P.S. In parts (a) and (b) the explanation would benefit if you clearly explained what physical process was considered, which assumptions were made etc. I.e. that it was enough to just consider the phase of a slow quasistatic motion of the piston (and why), that the system was assumed to be thermally isolated, so that from the energy conservation the temperature right before the piston was released was the same as the final equilibrium temperature etc. From your equations I see that you understand this all, but you really have to spend some time and transform your understanding into a clear write-up in future.



T

December 16, 2013 at 5:59 pm

Thank you so much for your helpful comment. I will try to revise the solution, especially the explanation in an understandable way.
