J03M.3

Solution to J03M.3

Based on the conservation of energy, we have:

$$\theta(r) = \int \frac{(l/r^2)du}{\sqrt{2m(E - U - \frac{l^2}{2mr^2})}} \tag{1}$$

Let $u = 1/r$, we have :

$$\theta(u) = \int \frac{ldu}{\sqrt{2m(E - U - \frac{l^2u^2}{2m})}} \tag{2}$$

(We drop the $\pm$, because it means little in our problem.)

Put $U = -C/2u^2$ into,

$$\theta(u) = \int \frac{ldu}{\sqrt{2m(E + (C/2 - \frac{l^2}{2m})u^2)}} \tag{3}$$

The value of this integrate depends on the value of $\epsilon = E/(2ml^2)$ and $\beta = mC/l^2 - 1$

and the eqn turns into:

$$\theta(u) = \int \frac{1du}{\sqrt{\epsilon + \beta u^2}} \tag{4}$$

Case 1 If $\epsilon > 0, \beta > 0$(i.e. $E > 0, mC/l^2 - 1 > 0$)

$$\theta(u) = \frac{1}{\sqrt{\beta}} \int \frac{du}{\sqrt{\epsilon/\beta + u^2)}} \tag{5}$$

$$\theta(u) - \theta(0) = \frac{1}{\sqrt{\beta}} acrsh(\sqrt{\beta/\epsilon} u) \tag{6}$$

i.e.

$$u = \sqrt{\epsilon/\beta} sh((\theta - \theta_0)\sqrt{\beta}) \tag{7}$$

Case 2 else if $\epsilon > 0, \beta < 0$ (i.e. $E > 0, mC/l^2 - 1 < 0$)

$$\theta(u) = \frac{1}{sqrt{-\beta}} \int \frac{du}{\sqrt{\epsilon/(-\beta) - u^2)}} \tag{8}$$

(Note now $-\beta > 0$)

$$\theta(u) - \theta(0) = \frac{1}{\sqrt{\beta}} \frac{|u|}{u} acrch(-\beta|u|/\epsilon) \tag{9}$$

Because u>0 is always true for our case,

$$u = \sqrt{\epsilon/(-\beta)} cosh((\theta - \theta_0)\sqrt{-\beta}) \tag{10}$$

Case 3 else if $\epsilon < 0, \beta < 0$ (i.e. $E < 0, mC/l^2 - 1 < 0$)

$$\theta(u) = \frac{1}{i\sqrt{-\beta}} \int \frac{du}{\sqrt{\epsilon/\beta + u^2)}} \tag{11}$$

So:

$$u = \sqrt{\epsilon/\beta} sinh(i(\theta - \theta_0)\sqrt{-\beta}) \qquad (12)$$

i.e.

$$u = \sqrt{\epsilon/\beta} sinh(i(\theta - \theta_0)\sqrt{\beta}) \qquad (13)$$

Case 4 else if $\epsilon < 0, \beta > 0$ (i.e. $E < 0, mC/l^2 - 1 < 0$)

Case 5 else if $\epsilon = 0, \beta > 0$ (i.e. $E = 0, mC/l^2 - 1 > 0$)

Case 6 else if $\epsilon = 0, \beta > 0$ (i.e. $E = 0, mC/l^2 - 1 > 0$)

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2 thoughts on “J03M.3”

Imaginary answer in Case 3 makes no sense, don't you think so?
In case 2 there should be an ordinary $\cos$.

There should be one more prameter for the orbit -- particle's total energy.
What you considered seems to be the case of zero energy.
You should consider non-zero total energy too.