J03M.1

Figure_1

Part a)

We can calculate θ_2 from conservation of energy and momentum parallel to the surface:
Conservation of energy: $E = \frac{1}{2}mv_1^2 = \ \frac{1}{2}mv_2^2 - V_0$

Conservation of momentum: $v_1 sin\theta_1 = v_2 sin\theta_2$

Thus:$sin\theta_2 = \sqrt{\frac{E}{E+V_0}} sin\theta_1 = \frac{sin\theta_1}{n}$ where $n^2 = \frac{E+V_0}{E}$

Part b)

From the diagram above it is straight forward to see that the total deflection is $2\alpha - 2\phi$(at each surface it is deflected by $\alpha - \phi$). Note that the second intersection with the boundary occurs at an angle of ϕ, as the triangle drawn is an isosceles triangle, and from symmetry we know that it must leave at an angle α with respect to the normal.

Now $sin\alpha = \frac{b}{R}$ from trigonometry.
We can also express ϕ in terms of α using the results from part a. So $\phi = arcsin(b/an)$with n defined as in part a.
Thus $\frac{\theta}{2} = arcsin(\frac{b}{R}) - arcsin(\frac{b}{Rn})$

From this we see that $\frac{d\theta}{db} = 2(\frac{1}{\sqrt{R^2 - b^2}} - \frac{1}{\sqrt{R^2 n^2 - b^2}})$

We are interested in $\frac{d\sigma}{d\Omega}$ and as such we wish to express $sin\theta$in terms of b. This can be done using double angle formulae:

$$sin\theta = sin(2\alpha)cos(2\phi) - sin(2\phi)cos(2\alpha) \tag{1}$$
$$= 2\,\frac{b}{nR}\left[n\sqrt{1-\frac{b^2}{R^2}}\,(1-\frac{2b^2}{R^2n^2}) - \sqrt{1-\frac{b^2}{n^2R^2}}\,(1-\frac{2b^2}{R^2})\right]$$

Collecting all these parts we find that:

$$\frac{d\sigma}{d\Omega} = \frac{nR}{4}\left[n\sqrt{1-\frac{b^2}{R^2}}\,(1-\frac{2b^2}{R^2n^2}) - \sqrt{1-\frac{b^2}{n^2R^2}}\,(1-\frac{2b^2}{R^2})\right]^{-1}((\frac{1}{\sqrt{R^2-b^2}} - \frac{1}{\sqrt{R^2n^2-b^2}})^{-1} \tag{2}$$

Alternatively the differential crosssection can be expressed as a function of θ using

$$b = \frac{ansin(\frac{\theta}{2})}{\sqrt{1+n^{-}2ncos(\frac{\theta}{2})}} \tag{3}$$

We get:

$$\frac{d\sigma}{d\Omega} = \frac{a^2n^2}{2(1+n^2-2ncos(\frac{\theta}{2}))}\left[\frac{cos(\frac{\theta}{2})}{2sin\theta} + \frac{n}{1+n^2-2ncos(\frac{\theta}{2})}\right] \tag{4}$$

One thought on “J03M.1”

October 8, 2013 at 5:26 pm

Looks good.
There is a typo in (3).
But otherwise everything seems correct.