To find $\theta_2$ we apply conservation of energy and momentum to $z < 0$ and $z > 0$:

$$E = E' - V_0$$  \hfill (1)

$$p \sin \theta_1 = p' \sin \theta_2.$$  \hfill (2)

Using these in addition to $E = \frac{p^2}{2m}$ and $E' = \frac{p'^2}{2m}$ we get

$$\frac{\sin \theta_1}{\sin \theta_2} = \sqrt{\frac{E'}{E}} = \sqrt{\frac{E + V_0}{E}}.$$  \hfill (3)

This is reminiscent of optical refraction with a refractive index.
The symmetry of the problem means that the triangle $ABC$ is isosceles. The previous result gives us

$$n = \sqrt{\frac{E + V_0}{E}},$$  \hspace{1cm} (4)

giving

$$\theta_2 = \arcsin \left( \frac{\sin \theta_1}{n} \right).$$  \hspace{1cm} (5)

2.

The symmetry of the problem means that the triangle $ABC$ is isosceles. The previous result gives us

$$\frac{\sin \theta_1}{\sin \theta_2} = n$$  \hspace{1cm} (6)

with $n$ given by (4). Geometry gives us the results

$$\theta = 2\phi$$  \hspace{1cm} (7)

$$\theta_1 = \phi + \theta_2$$  \hspace{1cm} (8)

$$b = R \sin \theta_1$$  \hspace{1cm} (9)

using which we immediately obtain

$$\phi = \frac{\theta}{2} = \arcsin \left( \frac{b}{R} \right) - \arcsin \left( \frac{b}{nR} \right).$$  \hspace{1cm} (10)

We also obtain $b(\theta)$ given by

$$b = \frac{nR \sin(\theta/2)}{\sqrt{1 - 2n \cos(\theta/2) + n^2}}.$$  \hspace{1cm} (11)

From (10),
We then obtain the differential cross section, given by
\[ \frac{d\theta}{db} = \frac{2}{R} \left( \frac{1}{\sqrt{1 - b^2/R^2}} - \frac{1}{\sqrt{1 - b^2/n^2 R^2}} \right). \] (12)

We then obtain the differential cross section, given by
\[ \frac{d\sigma}{d\Omega} = \frac{b}{\sin \theta} \frac{db}{d\theta} = \frac{b}{\sin \theta} \left[ \frac{2}{R} \left( \frac{1}{\sqrt{1 - b^2/R^2}} - \frac{1}{\sqrt{1 - b^2/n^2 R^2}} \right) \right]^{-1} \] (13)
where \( b \) is given by (11).

One thought on “J03M.1”

 October 8, 2013 at 3:09 am

Good solution.
Maybe (11) requires more explanation, at least one more step, to show how in an optimal way to solve (10) for \( b \).
Also check (12) – you made a small mistake when differentiating (10).