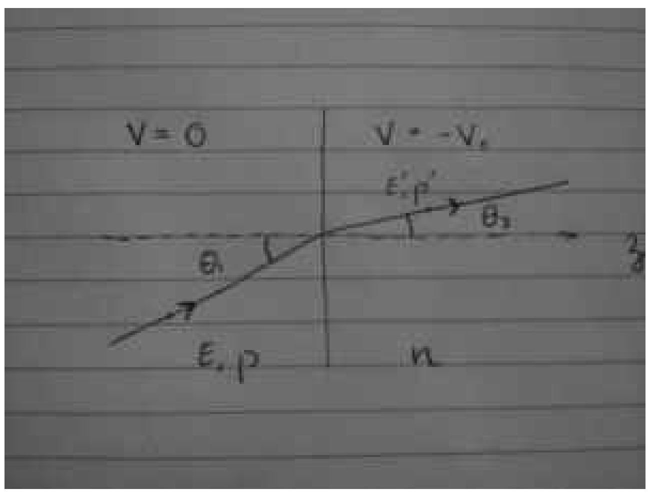
J03M.1

Solution to J03M.1 — Scattering from an Attractive Potential

1.



To find θ_2 we apply conservation of energy and momentum to $z < 0$ and $z > 0$:

$$E = E' - V_0 \tag{1}$$

$$p \sin\theta_1 = p' \sin\theta_2 . \tag{2}$$

Using these in addition to $E = \frac{p^2}{2m}$ and $E' = \frac{p'^2}{2m}$ we get

$$\frac{\sin\theta_1}{\sin\theta_2} = \sqrt{\frac{E'}{E}} = \sqrt{\frac{E+V_0}{E}} . \tag{3}$$

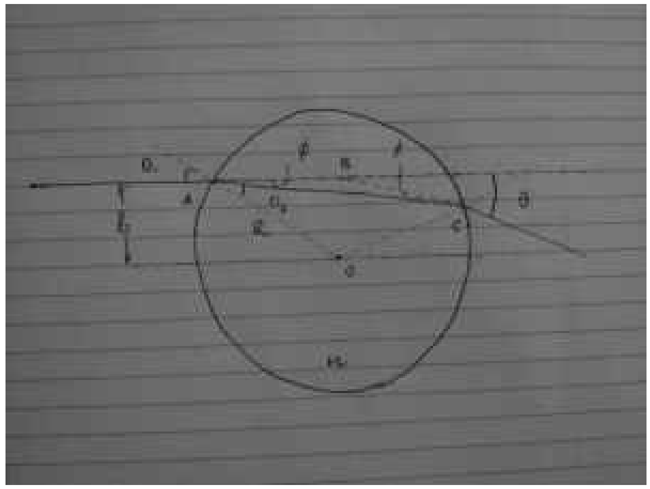
This is reminiscent of optical refraction with a refractive index

$$n = \sqrt{\frac{E + V_0}{E}}\,, \tag{4}$$

giving

$$\theta_2 = \arcsin\left(\frac{\sin\theta_1}{n}\right). \tag{5}$$

2.



The symmetry of the problem means that the triangle ABC is isosceles. The previous result gives us

$$\frac{\sin\theta_1}{\sin\theta_2} = n \tag{6}$$

with n given by (4). Geometry gives us the results

$$\theta = 2\phi \tag{7}$$

$$\theta_1 = \phi + \theta_2 \tag{8}$$

$$b = R\sin\theta_1 \tag{9}$$

using which we immediately obtain

$$\phi = \frac{\theta}{2} = \arcsin\left(\frac{b}{R}\right) - \arcsin\left(\frac{b}{nR}\right). \tag{10}$$

We also obtain $b(\theta)$ given by

$$b = \frac{nR\sin(\theta/2)}{\sqrt{1 - 2n\cos(\theta/2) + n^2}}. \tag{11}$$

From (10),

$$\frac{d\theta}{db} = \frac{2}{R}\left(\frac{1}{\sqrt{1-b^2/R^2}} - \frac{1}{\sqrt{1-b^2/n^2R^2}}\right). \tag{12}$$

We then obtain the differential cross section, given by

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\frac{db}{d\theta} = \frac{b}{\sin\theta}\left[\frac{2}{R}\left(\frac{1}{\sqrt{1-b^2/R^2}} - \frac{1}{\sqrt{1-b^2/n^2R^2}}\right)\right]^{-1} \tag{13}$$

where b is given by (11).

One thought on “J03M.1”

October 8, 2013 at 3:09 am

Good solution.
Maybe (11) requires more explanation, at least one more step, to show how in an optimal way to solve (10) for b.
Also check (12) -- you made a small mistake when differentiating (10).