1  J03E.2

1.1  a

These two charges have following potential

\[ V(x) = \frac{Q}{| \bar{a} - \bar{b} |} - \frac{\sqrt{\frac{b}{a}}Q}{| \bar{x} - \bar{a} |}, \]

where \( \bar{a} \) and \( \bar{b} \) are vector-positions of charges. Let us expand this formula and calculate scalar products

\[ V(x) = \frac{Q}{\sqrt{r^2 - 2rb \cos \phi + b^2}} - \frac{Q}{\sqrt{\frac{b}{a}\sqrt{r^2 - 2ra \cos \phi + a^2}}} = \]

\[ Q \left( \frac{1}{\sqrt{r^2 - 2rb \cos \phi + b^2}} - \frac{1}{\sqrt{\frac{br^2}{a} - 2rb \cos \phi + ab}} \right). \]

Here \( r = |x| \), we see that \( r = \sqrt{ab} \) makes \( V(x) = 0 \) irrespective of the value of angle \( \phi \). Hence we have an equipotential spherical surface \( V(x) = 0 \) of radius \( R = \sqrt{ab} \). It is worth noting that position \( a \) can be obtained from \( b \) by inversion \( a = \frac{R^2}{b} \), the \( a \) charge is \( q = -\frac{R}{b}Q \).

1.2  b

Electrical potential in this problem can be obtained by method of images, a partial justification of which was given in previous section. In order to make \( \phi(r, \theta z) \) potential equal to zero on a sphere of radius \( R \) we introduce an inverse conjugate charge \( -q' = -\frac{b}{a}q \) at \((\frac{R^2}{b} r_0, 0, \frac{R^2}{b} z_0)\). Next we have to make \( \phi(r, \theta z) \) on the plane \( z = 0 \). In order to achieve that we add to mirrored opposite charges \( -q \) and \( q' \) at positions \((r_0, 0, -z_0)\) and \((\frac{R^2}{b} r_0, 0, -\frac{R^2}{b} z_0)\) respectively. A final potential is a sum of one real charge and three images.

\[ \phi(r, \theta z) = \frac{q}{\sqrt{(r \cos \theta - r_0)^2 + r^2 \sin^2 \theta + (z - z_0)^2}} - \frac{q}{\sqrt{(r \cos \theta - r_0)^2 + r^2 \sin^2 \theta + (z + z_0)^2}} + \frac{\frac{R}{b} q}{\sqrt{(r \cos \theta - \frac{R^2}{b} r_0)^2 + r^2 \sin^2 \theta + (z - \frac{R^2}{b} z_0)^2}} + \frac{\frac{R}{b} q}{\sqrt{(r \cos \theta - \frac{R^2}{b} r_0)^2 + r^2 \sin^2 \theta + (z + \frac{R^2}{b} z_0)^2}}. \]
1.3 c

Applying Coulomb law directly we get

\[ F = -\frac{R}{z_0} q^2 \left( z_0 - \frac{R^2}{z_0} \right)^2 + \frac{R}{z_0} q^2 \left( z_0 + \frac{R^2}{z_0} \right)^2 - \frac{q^2}{4z_0^2}, \]  

(4)