J03E.1

We start by writing the Maxwell Equations:

$$\nabla \cdot \mathbf{D} = \rho_f$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}$$

In Aluminum we have:

$$\mathbf{D} = \varepsilon \mathbf{E}$$

$$\mathbf{B} = \mu \mathbf{H}$$

$$\mathbf{J} = \sigma \mathbf{E}$$

By taking the plane wave of the form $\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0 e^{i\mathbf{kr} - i\omega t}, \mathbf{H}(\mathbf{r}, t) = \mathbf{H}_0 e^{i\mathbf{kr} - i\omega t}$,

Then we have:

$$\mathbf{k} \cdot \mathbf{D} = 0$$

$$\mathbf{k} \cdot \mathbf{B} = 0$$

$$i\mathbf{k} \times \mathbf{E} = -i\omega\mu_0\mathbf{H}$$

$$i\mathbf{k} \times \mathbf{H} = \sigma\mathbf{E} + -i\omega\epsilon\mathbf{E}$$

By combining the last two equations we can get:

$$\mathbf{k}^2 = \omega^2\epsilon\mu_0 + i\sigma\omega\mu_0 \equiv n^2\,\frac{\omega^2}{c^2}$$

so for the first sub-problem a), we have the answer:

$$k = \omega\sqrt{\epsilon\mu_0 + i\,\frac{\sigma\mu}{\omega}}$$

for the second sub-problem b), we need to calculate the incident power reflected, due to the $\mathbf{E}$ is parallel to the plane of incidence, we have:

$$\frac{\mathbf{E}_r}{\mathbf{E}_i} = \frac{n_t - n_i}{n_t + n_i}$$

in which $n_t = c\sqrt{\epsilon\mu_0 + i\,\frac{\sigma\mu}{\omega}}$, $n_i = 1$,so energy reflection fraction can be written as:

$$r = |\,\frac{n_t - n_i}{n_t + n_i}\,|^2 = \frac{\sqrt{\frac{1+\sqrt{1+(\frac{\sigma}{\omega\epsilon})^2}}{2}} - 1}{?\sqrt{\frac{1+\sqrt{1+(\frac{\sigma}{\omega\epsilon})^2}}{2}} + 1}$$

the third sub-problem asks us about the emissivity \epsilon, due to the equilibrium requirement, the power emitted equals to the power absorbed. However, for the black-body, the incident power is total absorbed, to the normal emissivity equals to the ratio of power absorption. then we have:

$$\epsilon = t = 1 - r = \frac{2}{\sqrt{\frac{1+\sqrt{1+(\frac{\sigma}{\omega\epsilon})^2}}{2}} + 1} = 0.00106847$$

2 thoughts on "J03E.1"

M December 12, 2013 at 8:05 pm

Your solution looks correct.
But I think there are some typos in your formulas: your expression for $\mathbf{k}^2$ implies $n_t = c\sqrt{\epsilon\mu_0}\sqrt{1+i\frac{\sigma}{\omega\epsilon}}$ (rather than what you have), which however is consistent with your final formula for r if we take $\epsilon = \epsilon_0$.

T
December 16, 2013 at 6:04 pm

Thank you! The typo has been corrected.