Jan 2002 #2 (SM)

N nonrel. neutrons, magnetic moment μ_B, volume V, field H

a. $\bar{n}_s = \dfrac{1}{e^{\beta(\epsilon_s - \mu)} + 1}$ F.D. distribution, # of particles in state s with energy ϵ_s

neutron state given by both the spin state and translational state

$$\epsilon = \epsilon_{spin} + \epsilon_{trans}$$

spin up: $\epsilon_{spin} = -\mu_B H$ spin down: $\epsilon_{spin} = \mu_B H$

$$\epsilon_{trans} = \frac{\hbar^2 k^2}{2m} \quad \text{(nonrelativistic)}$$

For neutrons in the spin up (or down) state, any translational state can be occupied:

$$N^+ = \int_0^\infty f_{FD}(\epsilon_{spin} + \epsilon_{trans})\, \rho(\epsilon)\, d\epsilon_{trans}$$ $\rho(\epsilon)$ = density of states of translatational states

$\rho(\vec{k})d^3\vec{k} = \frac{V}{(2\pi)^3} d^3\vec{k}$ $\rho(k)dk = \frac{V}{2\pi^2} k^2 dk$ $\rho(\epsilon)d\epsilon = \rho(k(\epsilon)) \left|\frac{dk}{d\epsilon}\right| d\epsilon$

$k = \sqrt{\frac{2m}{\hbar^2}}\, \epsilon^{1/2}$ $\frac{dk}{d\epsilon} = \sqrt{\frac{m}{2\hbar^2}}\, \epsilon^{-1/2}$ $\rho(\epsilon) = \frac{V}{\pi^2} \frac{m}{\hbar^2} \epsilon \cdot \sqrt{\frac{m}{2\hbar^2}}\, \epsilon^{-1/2} = \frac{V}{\hbar^3 \pi^2} \frac{m^{3/2}}{\sqrt{2}} \epsilon^{1/2}$

$$N^+ = \frac{V m^{3/2}}{\sqrt{2}\hbar^3 \pi^2} \int_0^\infty \frac{1}{e^{\beta(-\mu_B H + \epsilon - \mu)} + 1} \epsilon^{1/2} d\epsilon$$

Similarly,

$$N^- = \frac{V m^{3/2}}{\sqrt{2}\hbar^3 \pi^2} \int_0^\infty \frac{1}{e^{\beta(\mu_B H + \epsilon - \mu)} + 1} \epsilon^{1/2} d\epsilon$$

b. $\mu \to \epsilon_F$ $T \to 0$ for spin up, if $\epsilon > \mu_B H + \mu$, $f_{FD} \to 0$

spin up: $$\int_0^{\epsilon_f + \mu_B H} \epsilon^{1/2} d\epsilon = \frac{2}{3}(\epsilon_f + \mu_B H)^{3/2}$$

spin down: $$\int_0^{\epsilon_f - \mu_B H} \epsilon^{1/2} d\epsilon = \frac{2}{3}(\epsilon_f - \mu_B H)^{3/2}$$

c. $$M = \frac{N^+ \mu_B - N^- \mu_B}{V} = \frac{\mu_B m^{3/2}}{\sqrt{2}\hbar^3 \pi^2} \cdot \frac{2}{3}\left[(\epsilon_f + \mu_B H)^{3/2} - (\epsilon_f - \mu_B H)^{3/2}\right]$$

ϵ_f determined by $N^+ + N^- = N$

$$\frac{Vm^{3/2}}{\sqrt{2}\hbar^3\pi^2}\cdot\frac{2}{3}\cdot\left[(\epsilon_f+\mu_B H)^{3/2}+(\epsilon_f-\mu_B H)^{3/2}\right]=N$$

d. $\mu_B H \ll \epsilon_f$ $\quad (\epsilon_f+\mu_B H)^{3/2}-(\epsilon_f-\mu_B H)^{3/2} = \epsilon_f^{3/2}\left[\left(1+\frac{\mu_B H}{\epsilon_f}\right)^{3/2}-\left(1-\frac{\mu_B H}{\epsilon_f}\right)^{3/2}\right]$

$$\approx \epsilon_f^{3/2}\left[\left(1+\frac{3}{2}\frac{\mu_B H}{\epsilon_f}\right)-1+\frac{3}{2}\frac{\mu_B H}{\epsilon_f}\right] \approx 3\epsilon_f^{1/2}\mu_B H$$

$$M = \frac{\sqrt{2}\,m^{3/2}}{\hbar^3\pi^2}\mu_B^2 H \epsilon_f^{1/2}$$

$$\chi = \frac{M}{H}$$