1 January 2002, Mechanics, Problem 2

$$\epsilon = 1 - \frac{2}{\frac{b}{a} + 1} = \frac{b/a - 1}{b/a + 1} \quad Eccentricity$$  \hfill (1)

I don’t know what they mean by characteristic radius.

$$\frac{GMm}{a^2} = \frac{mv_a^2}{a}$$

$$E = \frac{1}{2} \frac{mv_a^2}{a} - \frac{GMm}{a} = - \frac{mag}{2}$$  \hfill (2)

$$L = am\sqrt{ag}$$  \hfill (3)

$$v_a = \sqrt{ag}$$  \hfill (4)

$$v_b = a\sqrt{ag}$$  \hfill (5)

As the satellite passes by Mars, it seems that Mars gives it a boost equal to its own velocity, while changing the sign of the original velocity. Thus, the final velocity of the satellite is:

$$u = V_{Mars} - v_b$$

$$\frac{GM M_{Mars}}{b^2} = \frac{M_{Mars} V_{Mars}^2}{b} \rightarrow V_{Mars} = \sqrt{\frac{GM}{b}}$$

Now we plug this into the conservation of energy and angular momentum equation:

$$\frac{1}{2} mu^2 - \frac{GMm}{b} = \frac{1}{2} m (r^2 + r^2 \dot{\phi}^2) - \frac{GMm}{r}$$

$$mbu = mr^2 \dot{\phi}$$

Since they want some maximum distance, I assume that it’s not going to be infinity. Therefore, the maximum distance will occur at a point with \(\dot{r} = 0\). Plug that in to get:

$$\frac{1}{2} m (V_{Mars} - v_b)^2 - \frac{GMm}{b} = \frac{1}{2} mr_{\text{max}}^2 \left( \frac{bu}{r_{\text{max}}^2} \right)^2 - \frac{GMm}{r_{\text{max}}}$$

Solve for \(r_{\text{max}}\) to obtain:

$$r_{\text{max}} = \frac{g a^2}{u^2} \sqrt{1 + \left(1 - \frac{a}{b}\right)^4} - 1$$  \hfill (6)