# Flappy Toy

Fang Xie[1]

[1] *Department of Physics, Princeton University, Princeton NJ 08544, U.S.A.*

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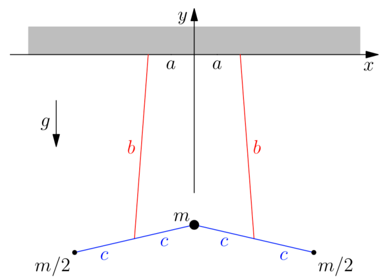
## PROBLEM

Deduce the frequency of small oscillations of the flapping toy shown in the figure below, supposing the central mass $m$ moves only vertically, and the motion of the others masses is only in the $x-y$ plane.




The flapping toy consists of the central mass $m$ connected by two massless rods of length $2c$ to two masses $m/2$. The centers of the two rods are suspended from a horizontal plane by massless strings of length $b$, with distance $2a$ between the upper points of suspension. Length $c$ is slightly larger than $a$.

## SOLUTION

First consider about the gravititional potential energy of the system

$$V = -mgh_1 - \frac{1}{2}mgh_2 - \frac{1}{2}mgh_2 = -2mgh$$

in which $h_1$ is the sidtance between the small particle with mass $m$ and the ceiling, $h_2$ is the distance between the particle with mass $m/2$ and the ceiling and $h = (h_1 + h_2)/2$ is the distance between the middle point of the rod and the ceiling. Now assume the angle between the rods and the $x$-axis is $\theta$, then we can get $h$ by the geometry relation:

$$h = \sqrt{b^2 - (c\cos\theta - a)^2}$$

Since this is a small oscillation, we know $\cos\theta \sim 1$ and then because $c \sim a$, we can get $(c\cos\theta - a) \ll b$. Then expand $h$ to the second order of $\cos\theta$, we get

$$h \sim \frac{ac}{b}\cos\theta - \frac{c^2}{2b}\cos^2\theta + \text{const.} \sim -\frac{c(a-c)}{2b}\theta^2 + \text{const.}$$

So up to an unimportant constant, the potential energy is

$$V = \frac{mgc(a-c)}{b}\theta^2$$

Then we derive the kinetic energy of this system:

$$T = \frac{1}{2} mc^2 \dot{\theta}^2 + 2\frac{1}{2}\left(\frac{m}{2}\right) c^2 \dot{\theta}^2 = mc^2 \dot{\theta}^2$$

We just neglected the contribution from the changing of $h$, because this contribution is at the order of $\theta^4$ at least. Now we get the Lagrangian as shown

$$L = mc^2 \dot{\theta}^2 - \frac{mgc(a-c)}{b} \theta^2$$

and from the equation of motion it is obvious to get the oscillation frequency

$$\omega = \sqrt{\frac{g}{bc}(a-c)}\,.$$