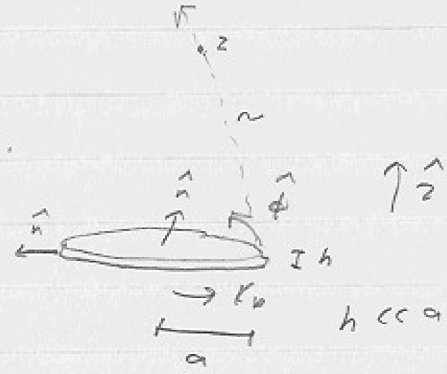
Redo Problem EM J02 #3



a) No free currents, but there are bound currents.

$J_b = \nabla \times \vec{m} = 0$

$K_b = m \times \hat{n} = m\hat{\phi}$ along outer edge

We have a ring of bound current $K_b = m\hat{\phi}$ along edge of disk. Because $h \ll a$, we can approximate this as a ring of current $I = hm$

Use Biot-Savart Law!

$$d\vec{l} = a\, d\phi\, \hat{\phi}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \oint \frac{d\vec{l} \times \hat{r}}{r^2}$$

$$\vec{B} = \frac{\mu_0}{4\pi} h m a \int_0^{2\pi} \frac{\hat{\phi} \times \hat{r}}{a^2 + z^2}\, d\phi$$

By symmetry, $\vec{B}$ along the axis will only be in the $\hat{z}$ direction. So take $\hat{z}$-component of $\hat{\phi} \times \hat{r}$

$$(\hat{\phi} \times \hat{r})_z = \cos\theta\, \hat{z} = \frac{a}{r}\hat{z} = \frac{a}{\sqrt{a^2+z^2}}\hat{z}$$

$$\vec{B}(z) = \frac{\mu_0 h m a}{4\pi} \frac{a}{(a^2+z^2)^{3/2}} \int_0^{2\pi} d\phi\, \hat{z}$$

$$= \frac{\cancel{2\pi}}{2\,\cancel{4\pi}} \frac{\mu_0 h m a^2}{(a^2+z^2)^{3/2}} \rightarrow \boxed{\vec{B}(z) = \frac{\mu_0 h m}{2} \frac{a^2}{(a^2+z^2)^{3/2}}\hat{z}}$$

b) Force on body w/ dipole moment m!

$$\vec{F} = \nabla(\vec{m}\cdot\vec{B}) \qquad \vec{m} = (\mu-1)V\vec{B}$$

$$\vec{F} = (\mu-1)V\nabla(\vec{B}\cdot\vec{B}) = (\mu-1)V\nabla B^2 = 2(\mu-1)VB\nabla B$$

$$\nabla B = \frac{dB}{dz}\hat{z} = \frac{\mu_0 h m a^2}{\cancel{2}}\frac{(-\frac{3}{2})\cancel{2}z}{(a^2+z^2)^{5/2}}\hat{z}$$

$$\nabla B = -\frac{3}{2}\frac{\mu_0 h m a^2}{(a^2+z^2)^{5/2}}z\hat{z}$$

$$\vec{F} = \cancel{2}(\mu-1)V\frac{\mu_0 h m}{\cancel{2}}\frac{a^2}{(a^2+z^2)^{3/2}}\left(-\frac{3}{2}\right)\mu_0 h m\frac{a^2}{(a^2+z^2)^{5/2}}z\hat{z}$$

$$\vec{F} = \frac{3(1-\mu)V(\mu_0 h m a^2)^2}{2}\frac{z}{(a^2+z^2)^4}\hat{z} \qquad \mu<1 \rightarrow F>0$$

Set $\vec{F} = k\vec{g} \rightarrow k = \dfrac{F}{g} \qquad \dfrac{dk}{dz} = 0$

$$\vec{F} = C_0\frac{z}{(a^2+z^2)^4} \rightarrow \frac{dF}{dz} = 0$$

$$\frac{1}{(a^2+z^2)^4} - \frac{4z(2z)}{(a^2+z^2)^5} = 0 \rightarrow \frac{1}{\cancel{(a^2+z^2)^4}} = \frac{8z^2}{(a^2+z^2)^{\cancel{5}}}$$

$$1 = \frac{8z^2}{a^2+z^2} \rightarrow a^2+z^2 = 8z^2$$

$$a^2 = 7z^2 \rightarrow z_0^2 = \frac{a^2}{7} \rightarrow \boxed{z_0 = \frac{a}{\sqrt{7}}}$$

$$\boxed{K_{max} = \frac{3}{2}\frac{(1-\mu)}{g}V(\mu_0 h m)\frac{1}{a^3}\frac{1}{\sqrt{7}}\left(\frac{7}{8}\right)^4}$$