1 January 2002, Electromagnetism, Problem 3

1.1 (a)

The problem of finding the magnetic field of a magnetized bar was done before. Refer to M04E3. The field is:

\[ B = \frac{B_0}{2} \left( -\frac{z}{\sqrt{z^2 + a^2}} + \frac{z + h}{\sqrt{(z + h)^2 + a^2}} \right) \hat{z} \]

where \( B_0 \) is given by:

\[ B_0 = \mu_0 K = \mu_0 m \]

Definition 1. \( f(z) = \frac{z}{\sqrt{z^2 + a^2}} \)

\[ B = \frac{\mu_0 m}{2} f'(z) h \hat{z} = \frac{\mu_0 m}{2} \frac{a^2}{(z^2 + a^2)^{3/2}} h \hat{z} \quad (1) \]

where I used the fact that \( h < < a \), and assumed that I only want the field for \( z > > h \).

1.2 (b)

The force on the frog is given by:

\[ F = (M \cdot \nabla) B = (\mu - 1)(\mu_0 m)^2 \pi b^3 \frac{h^2 a^4}{(z^2 + a^2)^2} 2z = kg \]

The maximum mass will correspond to a maximum of the force:

\[ \frac{dF}{dz} = (\mu - 1)(\mu_0 m)^2 \pi b^3 \frac{a^2 - 7z^2}{(a^2 + z^2)^5} h^2 a^4 2 = 0 \]

\[ z_0 = \frac{a}{\sqrt{7}} \quad (2) \]

We know this is a maximum because it is the only point where \( dF/dz=0 \), and at the ends \( z=0 \) and \( z=\infty \) the force is 0. The maximum mass is obtained by plugging in \( z_0 \) into the force equation:

\[ k_{max} = \frac{2(\mu - 1)\mu_0 m^2 \pi b^3 h^2}{g \sqrt{7} a^3} \left( \frac{7}{8} \right)^4 \quad (3) \]