1 January 2002, Electromagnetism, Problem 3

1.1 (a)

The problem of finding the magnetic field of a magnetized bar was done before. Refer to M04E3. The field is:

$$\boldsymbol{B} = \frac{B_0}{2}\left(-\frac{z}{\sqrt{z^2+a^2}} + \frac{z+h}{\sqrt{(z+h)^2+a^2}}\right)\hat{z}$$

where B_0 is given by:

$$B_0 = \mu_0 K = \mu_0 m$$

Definition 1. $f(z) = \frac{z}{\sqrt{z^2+a^2}}$

$$\boldsymbol{B} = \frac{\mu_0 m}{2} f'(z) h\hat{z} = \frac{\mu_0 m}{2}\frac{a^2}{(z^2+a^2)^{3/2}} h\hat{z} \tag{1}$$

where I used the fact that $h << a$, and assumed that I only want the field for $z >> h$.

1.2 (b)

The force on the frog is given by:

$$F = (\boldsymbol{M}\cdot\nabla)B = (\mu-1)(\mu_0 m)^2 \pi b^3 \frac{h^2 a^4}{(z^2+a^2)^4} 2z = kg$$

The maximum mass will correspond to a maximum of the force:

$$\frac{dF}{dz} = (\mu-1)(\mu_0 m)^2 \pi b^3 \frac{a^2-7z^2}{(a^2+z^2)^5} h^2 a^4 2 = 0$$

$$z_0 = a/\sqrt{7} \tag{2}$$

We know this is a maximum because it is the only point where dF/dz=0, and at the ends (z=0 and z=∞) the force is 0. The maximum mass is obtained by plugging in z_0 into the force equation:

$$k_{max} = \frac{2(\mu-1)\mu_0 m^2 \pi b^3 h^2}{g\sqrt{7}a^3}\left(\frac{7}{8}\right)^4 \tag{3}$$