1 January 2002, Electromagnetism, Problem 2

1.1 (a)

You have to solve Laplace's equation with azimuthal symmetry and no z-dependence:

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) = 0$$

Using boundary conditions leads to:

$$\boldsymbol{E}_{in} = -\frac{\lambda}{2\pi\epsilon_0\rho}\hat{\rho}$$
$$\boldsymbol{E}_{out} = \boldsymbol{0}$$

By Faraday's law:

$$2\pi a E_{ind} = -\pi a^2\frac{\partial B_{ex}}{\partial t}$$
$$\boldsymbol{E}_{ind} = -\frac{a}{2}\frac{\partial B_{ex}}{\partial t}\hat{\phi}$$
$$\boldsymbol{F}_{ind} = -\lambda l\frac{a}{2}\frac{\partial B_{ex}}{\partial t}\hat{\phi}$$
$$\boldsymbol{\tau}_{ind} = -\lambda l\frac{a^2}{2}\frac{\partial B_{ex}}{\partial t}\hat{\phi} = Il\dot{\boldsymbol{\omega}}$$
$$I\omega = -\lambda\frac{a^2}{2}(B_f - B_{ex})$$
$$B_f = \mu_0 K = \mu_0\sigma v = \mu_0\frac{\lambda}{2\pi a}a\omega = \frac{\mu_0\lambda\omega}{2\pi}$$
$$\Delta B = \frac{\mu_0\lambda\omega}{2\pi} - B_{ex}$$
$$\omega = -\lambda\frac{a^2}{2I}\left(\frac{\mu_0\lambda\omega}{2\pi} - B_{ex}\right)$$
$$\omega = \frac{\lambda a^2 B_{ex}}{2\left(I + \frac{\lambda^2 a^2\mu_0^2}{4\pi}\right)} \tag{1}$$

Anatoly Dimarsky obtained a slightly different solution, with aT in the numerator instead of a^2. But I don't think it's right, because there is no way that I can obtain a T from anywhere.