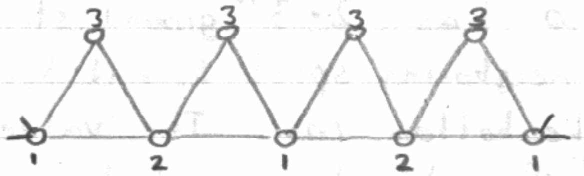
Seth Dorfman Prelims December 19, 2005

January 2001 SM

3)a.



$H = J \sum_{<ij>} S_i S_j$

$H_1 = J S_1 (2S_2 + 2S_3) = M_1 S_1$ $\qquad M_1 = 2J(\langle S_2\rangle + \cancel{\langle S_3\rangle}^{0})$

$H_2 = J S_2 (2S_1 + 2S_3) \underset{\uparrow}{\approx} M_2 S_2$ $\qquad M_2 = 2J(\langle S_1\rangle + \cancel{\langle S_3\rangle}^{0})$

mean field approx

$Z_1 = e^{-\beta M_1} + e^{\beta M_1} \qquad Z_2 = e^{-\beta M_2} + e^{\beta M_2}$

$\langle S_1\rangle = \frac{e^{-\beta M_1} - e^{\beta M_1}}{e^{-\beta M_1} + e^{\beta M_1}}$

$\langle S_1\rangle = -\tanh(\beta M_1) \qquad \langle S_2\rangle = -\tanh(\beta M_2)$

$= -\tanh(\beta 2J\langle S_2\rangle)$

$= -\tanh(-2\beta J \tanh(\beta M_2))$

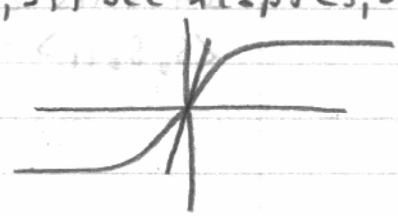
$= -\tanh(-2\beta J \tanh(2\beta J\langle S_1\rangle))$

$\langle S_1\rangle = \tanh(2\beta J \tanh(2\beta J\langle S_1\rangle))$

$\frac{\partial\langle S_1\rangle}{\partial\langle S_1\rangle} = \mathrm{sech}^2(2\beta J \tanh(2\beta J\langle S_1\rangle)) \cdot 2\beta J\, \mathrm{sech}^2(2\beta J\langle S_1\rangle)\, 2\beta J$

$= (2\beta J)^2\, \mathrm{sech}^2(2\beta J \tanh(2\beta J\langle S_1\rangle))\, \mathrm{sech}^2(2\beta J\langle S_1\rangle)$

$\left.\frac{\partial\langle S_1\rangle}{\partial\langle S_1\rangle}\right|_{\langle S_1\rangle = 0} = (2\beta J)^2$



At T_c: $1 = (2\beta_c J)^2$

$2J = \frac{1}{\beta_c}$

$T_c = \frac{2J}{k_B}$

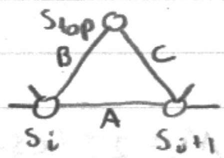
b. $2^3 = 8$ possibilities

two (all ↑ + all ↓) are an excited state

⇒ 6 possible ground states

c. Each triangle must have at least one "maximum energy bond" in which the spins on either side are the same. The ground state has exactly one such bond per triangle. There are thus 3^N ways to pick these bonds and two ways to fill in the spins once the "maximum energy bonds" are chosen. Thus a system of N triangles at $T=0$ has $2 \cdot 3^N$ ground states.

d. Consider two neigboring spins S_i and S_{i+1} from



the bottom row. The value of $S_i \cdot S_{i+1}$ depends on which bond in the triangle containing both spins is the "maximum energy bond"

Maximum Energy Bond	$S_i \cdot S_{i+1}$	Probability
A	1	$\frac{1}{3}$
B	-1	$\frac{1}{3}$
C	-1	$\frac{1}{3}$

$\langle S_i S_{i+1} \rangle = 1 \cdot \frac{1}{3} - 1 \cdot \frac{1}{3} - 1 \cdot \frac{1}{3} = -\frac{1}{3}$

Thus for two spins a distance $|j-i|$ from each other:

$\langle S_i S_j \rangle = \left(\frac{-1}{3}\right)^{|j-i|}$

This is not consistant with part (a) where B or C is always chosen to be the maximum energy bond. In this case:

$\langle S_i S_{i+1} \rangle = -1$ and $\langle S_i S_j \rangle = (-1)^{|j-i|}$