Seth Dorfman   Prelims   December 19, 2005

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2) a. $Z_{gc}^{(1)} = \sum_{\text{states}} e^{-\beta(E-\mu)N}$

$$= \left(\underset{\substack{\uparrow \\ \text{unbound}}}{1} + \underset{\substack{\uparrow \\ \text{bound to } A}}{e^{-\beta(-E_{Ac}-\mu_A)}} + \underset{\substack{\uparrow \\ \text{bound to } B}}{e^{-\beta(-E_{Bc}-\mu_B)}}\right)$$

$$Z_{gc} = \left(1 + e^{\beta(E_{Ac}+\mu_A)} + e^{\beta(E_{Bc}+\mu_B)}\right)^{N_c}$$

$$f_A = \frac{e^{\beta(E_{Ac}+\mu_A)}}{Z_{gc}} = \frac{e^{\beta(E_{Ac}+\mu_A)}}{1 + e^{\beta(E_{Ac}+\mu_A)} + e^{\beta(E_{Bc}+\mu_B)}}$$

$$f_B = \frac{e^{\beta(E_{Bc}+\mu_B)}}{Z_{gc}} = \frac{e^{\beta(E_{Bc}+\mu_B)}}{1 + e^{\beta(E_{Ac}+\mu_A)} + e^{\beta(E_{Bc}+\mu_B)}}$$

b. In the absence of B ($\mu_B \to -\infty$), $f_A = 1$

$\Rightarrow \mu_A \gg \varepsilon_{Ac}$

$$\therefore f_A \approx \frac{e^{\beta(E_{Ac}+\mu_A)}}{e^{\beta(E_{Ac}+\mu_A)} + e^{\beta(E_{Bc}+\mu_B)}} = \frac{1}{1 + e^{\beta(E_{Bc}-E_{Ac}+\mu_B-\mu_A)}}$$

Treating the A and B molecules clasically:

$n_A \approx e^{\beta\mu_A}$, $n_B \approx e^{\beta\mu_B}$

$$\Rightarrow f_A = \frac{1}{1 + \frac{n_B}{n_A} e^{\beta(E_{Bc}-E_{Ac})}}$$

c. $\frac{n_B}{n_A} = 0.01 \Rightarrow f_A = 0.1 = \frac{1}{1 + 0.01\, e^{\beta(E_{Bc}-E_{Ac})}}$

$T = 300\,K$

$$10 = 1 + 0.01\, e^{\beta(E_{Bc}-E_{Ac})}$$

$$900 = e^{\beta(E_{Bc}-E_{Ac})}$$

$$\beta(\varepsilon_{Bc}-\varepsilon_{Ac}) = \ln(30^2) = 2\ln(30)$$

$$\varepsilon_{Bc}-\varepsilon_{Ac} = 2k_B T \ln(30)$$

$$\varepsilon_{Bc}-\varepsilon_{Ac} = 2\,(8.62\times10^{-5}\,\tfrac{eV}{K})(300\,K)\ln(30$$

$$= 0.18\,eV$$