Seth Dorfman Prelims December 19, 2005

January 2001 QM

1) Let $\ell_0 = \sqrt{\frac{2\hbar}{M_1 \omega}}$ $\hat{x}_1 = \frac{\ell_0}{2}(a^+ + a)$ $V' = \lambda \delta(x_1 - x_2)$

$|\text{initial}\rangle = |p\rangle \otimes |0\rangle$ $|p\rangle = \frac{1}{\sqrt{L}} e^{ipx}$

$|\text{final}\rangle = |p^*\rangle \otimes |1\rangle$ $|p^*\rangle = \frac{1}{\sqrt{L^*}} e^{ip^*x}$

Energy conservation: $\frac{p^{*2}}{2m_2} = \frac{p^2}{2m_2} - \hbar\omega$

$$p^{*2} = p^2 - 2m_2\hbar\omega$$

$$p^* = \pm\sqrt{p^2 - 2m_2\hbar\omega}$$

$\langle \text{final}|V'|\text{initial}\rangle = \langle 1, p^*| \lambda\delta(x_1 - x_2)| 0, p\rangle$

$= \langle 1| \int dx_2 \frac{1}{\sqrt{L^*}} e^{-ip^*x_2} \lambda\delta(x_1 - x_2) \frac{1}{\sqrt{L}} e^{ipx_2} |0\rangle$

$= \langle 1| \frac{\lambda}{\sqrt{LL^*}} e^{i(p-p^*)x_1} |0\rangle$

$= \frac{\lambda}{\sqrt{LL^*}} \langle 1| e^{i(p-p^*)\ell_0(a^+ + a)/2} |0\rangle$

Let $z = \frac{i(p-p^*)\ell_0}{2}$ $|z|^2 = (p-p^*)^2 \frac{\ell_0^2}{4} = \frac{(p-p^*)^2\hbar}{2M_1\omega}$

$= \frac{\lambda}{\sqrt{LL^*}} \langle 1| e^{za^+ - \bar{z}a} |0\rangle$

$= \frac{\lambda}{\sqrt{LL^*}} \langle 1 | z\rangle$

Where $|z\rangle = e^{-|z|^2/2} \sum_n \frac{z^n}{n!} |n\rangle$ is a coherent state

$\therefore \langle \text{final}|V'|\text{initial}\rangle = \frac{\lambda}{\sqrt{LL^*}} \langle 1| e^{-|z|^2/2} \sum_n \frac{z^n}{n!} |n\rangle$

$= \frac{\lambda}{\sqrt{LL^*}} e^{-|z|^2/2} z$

$|\langle \text{final}|V'|\text{initial}\rangle|^2 = \frac{\lambda^2}{LL^*} e^{-|z|^2} |z|^2$

The probability of a transition can now be calculated to first order in pertubation theory:

$P^{sys}_{0\to 1}(t) = \frac{1}{\hbar^2} \left| \int_0^t e^{i(E_1 - E_0)t'/\hbar} \langle \text{final}|V'|\text{initial}\rangle \right|^2$

Since the energy of the <u>system</u> is conserved $E_1 - E_0 = 0$

$P_{0\to 1} = \frac{1}{\hbar^2} |t \langle \text{final}|V'|\text{initial}\rangle|^2$

$= \left(\frac{\lambda t}{\hbar\sqrt{LL^*}}\right)^2 e^{-|z|^2} |z|^2$

$= \left(\frac{\lambda}{\hbar}\right)^2 \frac{1}{vv^*} e^{-|z|^2} |z|^2$

Where $v = \frac{L}{t}$ and $v^* = \frac{L^*}{t}$ are the initial and final veclocities (of particle two)

Now, since there are two possible final values of p^* (as seen above) particle two may scatter in two different directions. Each possibility must be taken into account in $P_{0\to 1}$:

Let $p_f = \sqrt{p^2 - 2m_2\hbar\omega}$, $p_f^- = -\sqrt{p^2 - 2m_2\hbar\omega}$

$v_f = \frac{p_f}{m_2}$ $\qquad v_f^- = \frac{p_f^-}{m_2} = -\frac{p_f}{m_2}$ $\qquad v = \frac{p}{m_2}$

$$P_{0\to1} = \left(\frac{\lambda}{\hbar}\right)^2 \cdot \frac{1}{v}\left[\frac{1}{v_f}\frac{(p-p_f)^2\hbar}{2M_1\omega}e^{-(p-p_f)^2\hbar/2M_1\omega} + \frac{1}{v_f^-}\frac{(p-p_f^-)^2\hbar}{2M_1\omega}e^{-(p-p_f^-)^2\hbar/2M_1\omega}\right]$$

$$= \left(\frac{\lambda}{\hbar}\right)^2\frac{m_2}{p}\frac{m_2\hbar}{2M_1\omega}\left[\frac{1}{p_f}(p-p_f)^2 e^{-(p-p_f)^2\hbar/2M_1\omega} - \frac{1}{p_f}(p+p_f)^2 e^{-(p+p_f)^2\hbar/2M_1\omega}\right]$$

$$P_{0\to1} = \frac{\lambda^2 m_2^2}{2M_1\hbar\omega\, p\, p_f}\left[(p-p_f)^2 e^{-(p-p_f)^2\hbar/2M_1\omega} - (p+p_f)^2 e^{-(p+p_f)^2\hbar/2M_1\omega}\right]$$

where $p_f = \sqrt{p^2 - 2m_2\hbar\omega}$