# J01M.3

## Solution to J01m.3— Free Precession of a Planet-2

Euler's equations,

$$\boldsymbol{N} = \frac{d\boldsymbol{L}}{dt} + \boldsymbol{\omega} \times \boldsymbol{L} \tag{1}$$
$$\boldsymbol{L} = \boldsymbol{I} \cdot \boldsymbol{\omega} \tag{2}$$

That is

$$\begin{cases} I_1\dot{\omega_1} + (I_3 - I_2)\omega_2\omega_3 & = 0 \\ I_2\dot{\omega_2} + (I_1 - I_3)\omega_1\omega_3 & = 0 \\ I_3\dot{\omega_3} + (I_2 - I_1)\omega_1\omega_2 & = 0 \end{cases} \tag{3}$$

As $I_1 = I_2 = I_E,\, I_3 = I_p$
$I_3\dot{\omega_3} = 0$
$\omega_3 = const$
Thus

$$\dot{\omega_1} + \Omega\dot{\omega_2} = 0 \tag{4}$$
$$\dot{\omega_2} - \Omega\dot{\omega_1} = 0 \qquad \text{where} \quad \Omega = \frac{I_p - I_E}{I_E}\omega_3 = \epsilon\omega_3 \tag{5}$$

We can easily deduce that $\frac{I_p - I_E}{I_E} = \epsilon$ from the given condition $\frac{I_p - I_E}{I_p} = \epsilon$
Then

$$\omega_1 = \omega_0 cos(\Omega t + \alpha_0) \tag{6}$$
$$\omega_2 = \omega_0 sin(\Omega t + \alpha_0) \tag{7}$$

Thus, the $\Omega$ introduced here is the angular frequency of free precession.
$\Omega = \epsilon \omega cos \alpha$
where $\alpha$ is
angle between symmetry axis and axis of rotation.

For free precession of the earth, $\alpha = 0.2$ sec of arc, $\epsilon = 0.00327$, the preriod of precession is

$$\frac{2\pi}{\Omega} \approx 305 \, \text{days} \tag{8}$$

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## One thought on “J01M.3”

October 17, 2013 at 6:29 pm

Good.
You say "We can easily deduce that $\frac{I_P - I_E}{I_E} = \epsilon$ from the given condition $\frac{I_P - I_E}{I_E} = \epsilon$ " -- do you mean for small $\epsilon$?

Also, your equation $\Omega = \epsilon \omega \cos \alpha$ needs some derivation, I'd say so.

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