J01M.3

In the body frame, total angular momentum of the object J is

$$\vec{J} = I_x \omega_x \overrightarrow{e_x} + I_y \omega_y \overrightarrow{e_y} + I_z \omega_z \overrightarrow{e_z}$$

Given

$$\begin{cases} \dfrac{\mathrm{d}\vec{e}_x}{\mathrm{d}t} = \vec{\omega} \times \vec{e}_x = \omega_z \vec{e}_y - \omega_y \vec{e}_z \\ \dfrac{\mathrm{d}\vec{e}_y}{\mathrm{d}t} = \vec{\omega} \times \vec{e}_y = \omega_x \vec{e}_z - \omega_z \vec{e}_x \\ \dfrac{\mathrm{d}\vec{e}_z}{\mathrm{d}t} = \vec{\omega} \times \vec{e}_z = \omega_y \vec{e}_x - \omega_x \vec{e}_y \end{cases}$$

Hence,

$$\frac{\mathrm{d}\vec{J}}{\mathrm{d}t} = I_x \dot{\omega}_x \overrightarrow{e_x} + I_x \omega_x (\omega_z \vec{e}_y - \omega_y \vec{e}_z) + I_y \dot{\omega}_y \overrightarrow{e_y} + I_y \omega_y (\omega_x \vec{e}_z - \omega_z \vec{e}_x) + I_z \dot{\omega}_z \overrightarrow{e_z} \\ + I_z \omega_z (\omega_y \vec{e}_x - \omega_x \vec{e}_y) = \left[I_x \dot{\omega}_x - (I_y - I_z) \omega_y \omega_z \ \right] \overrightarrow{e_x} \\ + \left[I_y \dot{\omega}_y - (I_z - I_x) \omega_z \omega_x \ \right] \overrightarrow{e_y} + \left[I_z \dot{\omega}_z - (I_x - I_y) \omega_x \omega_y \ \right] \overrightarrow{e_z}$$

Also, because

$$\frac{\mathrm{d}\vec{J}}{\mathrm{d}t} = N_x \overrightarrow{e_x} + N_y \overrightarrow{e_y} + N_z \overrightarrow{e_z}$$

We get

\left\{

$$\begin{aligned} I_x\dot{\omega}_x - (I_y - I_z)\omega_y\omega_z &= N_x \\ I_y\dot{\omega}_x - (I_z - I_x)\omega_z\omega_x &= N_y \\ I_z\dot{\omega}_x - (I_x - I_y)\omega_x\omega_y &= N_z \end{aligned}$$

\right.

In this question $\vec{N} = 0$ and $I_x = I_y = I_E$, $I_z = I_P$, $(I_P - I_E)/I_P = \varepsilon$

So

$$I_P\dot{\omega}_z = 0 \quad \Rightarrow \omega_z = Constant = \omega$$

And

$$\begin{cases} \dot{\omega}_x + \dfrac{I_P - I_E}{I_E}\omega\omega_y = 0 \\ \dot{\omega}_y - \dfrac{I_P - I_E}{I_E}\omega\omega_x = 0 \end{cases}$$

Because ϵ very small which means $I_P \approx I_E$, $\frac{I_P - I_E}{I_P} \approx \frac{I_P - I_E}{I_E} = \epsilon$

Define $\Omega = \frac{I_P - I_E}{I_E}\omega = \epsilon\omega$

$$\begin{cases} \omega_x = Acos(\Omega t + \theta) \\ \omega_y = Asin(\Omega t + \theta) \end{cases}$$

We can see that in body frame, the rotation axis rotate around z axis with angular frequency Ω. Hence, the angular frequency of precession is $\epsilon\omega$