J01M.2 - Free Precession of a Planet - 1

Suppose that the density ρ of the object is uniform, and that its shape can be determined by the condition of hydrostatic equilibrium. Deduce an expression for the (small) quantity $\epsilon(\omega, M, r_P)$ that relates the equatorial radius r_E to the polar radius r_P by the form $r_E = r_P(1+\epsilon)$, where $M \approx 4\pi\rho r_P^3/3$ is the mass of the object.

Let F_P be the force due to the hydrostatic pressure.
Balancing the forces at the poles, we get

$$F_P - \frac{GMm}{r_P^2} = 0 \tag{1}$$

At the equator we have,

$$F_P - \frac{GMm}{r_E^2} - m\omega^2 r_E = 0 \tag{2}$$

Since ρ is uniform, m at both the poles and the equator can be thought of as ρdV and are thus equal. Since we are in an equilibrium state, we can equate F_P at the poles and at the equator, thus obtaining the equation

$$\frac{GM}{r_P^2} - \frac{GM}{r_E^2} - \omega^2 r_E = 0 \tag{3}$$

$$\frac{GM}{r_P^2}\left(1 - \frac{r_P^2}{r_E^2}\right) = \omega^2 r_E \tag{4}$$

Substituting $r_E = r_P(1+\epsilon)$ and solving for ϵ, we get

$$\epsilon = \frac{1}{\frac{2GM}{\omega^2 r_P^3} - 1} \tag{5}$$

One thought on “J01M.2 - Free Precession of a Planet - 1”

The pressure vanishes at the surface of the planet for an obvious reason (otherwise it would push the outer infinitely small layer of the liquid outside the planet with an infinite acceleration). So it is not clear what you mean by $\vec{F}_{pressure}$.