The hydrostatic equilibrium condition is that for static fluid, the potential energy along its surface should be a constant.

In this problem, we work in the rigid body coordinate system. Assume the rotating axis is along z direction. Since $\epsilon(\omega, M, r_P)$ is a small quantity, we approximate the potential energy due to gravity is $V_G = -\frac{GM}{r_P}$. The potential energy due to centrifugal force is $V_C = -\frac{1}{2} \omega^2 r$, where $r = \sqrt{x^2 + y^2}$.

The total potential energy on the pole should be the same as that on the equator. So,

$$-\frac{GM}{r_E} - \frac{1}{2} \omega^2 r_E^2 = -\frac{GM}{r_P}$$

(1)

expand $1/r_E = (1 - \epsilon)/r_P$, and $r_E = (1 + \epsilon)r_P$, we get

$$\epsilon = \frac{\omega^2 r_P^2}{2GM/r_P - 2\omega^2 r_P^2}$$

(2)

2 thoughts on “J01M.2”
Otherwise OK.

Why is it a good approximation?