J01M.2

The hydrostatic equilibrium condition is that for static fluid, the potential energy along its surface should be a constant.

In this problem, we work in the rigid body coordinate system. Assume the rotating axis is along z direction. Since $\epsilon(\omega, M, r_P)$ is a small quantity, we approximate the potential energy due to gravity is $V_G = -\frac{GM\rho}{r_P}$. The potential energy due to centrifugal force is $V_C = -\frac{1}{2}\rho\omega^2 r$, where $r = \sqrt{x^2+y^2}$.

The total potential energy on the pole should be the same as that on the equator. So,

$$-\frac{GM}{r_E} - \frac{1}{2}\omega^2 r_E^2 = -\frac{GM}{r_P} \tag{1}$$

expand $1/r_E = (1-\epsilon)/r_P$, and $r_E = (1+\epsilon)r_P$, we get

$$\epsilon = \frac{\omega^2 r_P^2}{2GM/r_P - 2\omega^2 r_P^2} \tag{2}$$

2 thoughts on “J01M.2”

December 3, 2013 at 4:33 am

Otherwise OK.

Why is it a good approximation?
