Consider a single frequency $\omega$ in the three regions we have:

$$E_i = E_0 e^{i(\omega t - kz)} \quad \text{for} \quad z < 0$$

$$E_1 = E_1 e^{i(\omega t - k'z)} \quad \text{for} \quad 0 < z < a$$

$$E_2 = E_2 e^{i(\omega t - kz)} \quad \text{for} \quad a < z$$

where $k' = \frac{\omega}{c} = \omega_0 + \frac{d\omega_0(\omega)}{d\omega} \left|_{\omega_0} \right. (\omega - \omega_0)$.

From continuity of $E$ parallel:

$$E_i(t, 0) = E_1(t, 0)$$

$$E_1 e^{-ik'a} = E_2 e^{-ika}$$

Now we wish to work out the frequency profile of the wave, to do this we consider the wave at $z=0$

$$E(\omega) = \int \frac{dt}{\sqrt{2\pi}} E(t, 0) e^{-i\omega t}$$

We can then match the different frequency components of the wave at the boundary, using the above boundary conditions, to deduce how the wave propagates in the medium (and by applying a factor of $e^{-ikz}$ to calculate the waves spatial propagation).

Using this we can calculate the wave in the medium as follows:
\[ E_1(t, z) = \int \frac{d\omega}{\sqrt{2\pi}} E_1(\omega) e^{i\omega t - ik'z} \]

\[ E_1(t, z) = \int \frac{d\omega}{\sqrt{2\pi}} E_0(\omega) e^{i\omega t - ik'z} \]

Thus the wave propagates with the same profile but now with a phase and a different velocity. An identical process can be performed for the wave after it has left the medium:

\[ E_1(t, z) = \int dt' e^{-i\omega_0(1 - \frac{d\omega n(\omega)}{d\omega}|_{\omega_0})} E(t', 0) e^{-i\omega t} e^{i\omega t - i\frac{(\omega_0 + \frac{d\omega n(\omega)}{d\omega}|_{\omega_0} (\omega_0 - \omega_0))}{c}z} \]

\[ E_1(t, z) = \int dt' e^{-i\omega_0(1 - \frac{d\omega n(\omega)}{d\omega}|_{\omega_0})} z E(t', 0) \delta(t - t') - \frac{(\omega_0 + \frac{d\omega n(\omega)}{d\omega}|_{\omega_0} (\omega_0 - \omega_0))}{c}z \]

\begin{equation}
E_1(t, z) = e^{\omega_0(1 - \frac{d\omega n(\omega)}{d\omega}|_{\omega_0})} f(t - \frac{(\omega_0 + \frac{d\omega n(\omega)}{d\omega}|_{\omega_0} (\omega_0 - \omega_0))}{c}z) \end{equation}

\end{equation}

Thus the wave propagates with the same profile but now with a phase and a different velocity. An identical process can be performed for the wave after it has left the medium:

\[ E_2(t, z) = \int \frac{d\omega}{\sqrt{2\pi}} E_2(\omega) e^{i\omega t - ikz} \]

\[ E_2(t, z) = \int \frac{d\omega}{\sqrt{2\pi}} E_0(\omega) e^{i(k-k')a} e^{i\omega t - ikz} \]

\[ E_2(t, z) = \int \int \frac{dt'}{\sqrt{2\pi}} \frac{d\omega}{\sqrt{2\pi}} E_0(t', 0) e^{-i\omega t'} e^{i(k-k')a} e^{i\omega t - ikz} \]

\[ E_2(t, z) = e^{-i\omega_0(1 - \frac{d\omega n(\omega)}{d\omega}|_{\omega_0})a} f(t - \frac{\omega_0}{c}a - \frac{z}{c}) \]

When \( \frac{d\omega n(\omega)}{d\omega}|_{\omega_0} \) is less then zero (i.e. the group velocity is less than zero) the wave appears to jump in time!!