Relativistic cyclotron motion:

\[
\frac{d\vec{p}}{dt} = \frac{q}{c} \vec{v} \times \vec{B} \quad \text{For pure circular motion at constant speed, } V = \text{constant} \]

⇒ some solution as non-relativistic cyclotron motion with \( m \rightarrow m' \)

\[
\Omega = \frac{eB}{m'} \quad |\Omega| = \frac{eB}{m'c} \]

\[
R = \frac{V}{\Omega} = \frac{c}{eB} \quad \frac{p}{eB} \quad \text{magnitude of } \vec{p} \]

The problem is asking what the condition on \( B \) is to make \( R \) a constant:

⇒ \[
\frac{dR}{dt} = 0 \quad \frac{1}{B} \frac{dp}{dt} = \frac{p}{B^2} \frac{dB}{dt} = 0 \quad \frac{B}{p} \frac{dp}{dt} = \frac{dB}{dt} \]

\( dp/\text{dt} \) changes magnitude because of an inductive electric field, not the magnetic field directly:

\[
\frac{dp}{dt} = -eE \]

\( E_y \) is determined by Faraday's law:

\[
\oint \mathbf{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int \mathbf{B} \cdot d\mathbf{A} \]

\[
2\pi RE = -\frac{\partial E}{\partial t} = \frac{\partial B}{\partial x} \frac{dx}{dt} \]

\[
E = \frac{R}{2c} \frac{\partial B}{\partial x} \]

⇒ \[
\frac{dp}{dt} = \frac{eR}{2c} \frac{\partial B}{\partial x} \]

⇒ \[
\frac{eB}{2c} \frac{\partial B}{\partial x} = \frac{dB}{dt} \quad R = \frac{c}{eB} \]

⇒ \[
\frac{eB}{2c} \frac{\partial B}{\partial x} = \frac{dB}{dt} \]

⇒ \[
\frac{1}{c} \frac{\partial B}{\partial t} = \frac{dB}{dt} \]

\[
\frac{d}{dt} (B - B_0) = 0 \quad B = B_0 + \text{const} \]
maximum energy: power in = power radiated

\[ P_{\text{rad}} = \frac{2}{3} \frac{e^2}{m^2c^3} \left[ \left( \frac{\Delta \Phi}{\Delta t} \right)^2 - \frac{1}{c^2} \left( \frac{d \Phi}{dt} \right)^2 \right] \]

For relativistic circular motion, \[ |\frac{d \Phi}{dt}| = \gamma \Omega |\beta| \gg \frac{e E}{m c} \quad |\beta| = \frac{m v}{c} \]

\[ P_{\text{rad}} = \frac{2}{3} \frac{e^2}{m^2c^3} \gamma^2 \Omega^2 \beta^2 \gamma^4 v^2 = \frac{2}{3} \frac{e^2 \Omega^2 \gamma^2 v^2}{c^5} \]

\[ Q = \frac{1}{R} \quad \text{or} \quad Q = e \beta \]

\[ r = m \beta c \]

ultra-relativistic: \[ \gamma \gg 1 \quad \Rightarrow \quad R = \frac{e E}{e \beta} \quad (E \gg m^2 c) \quad (\text{energy } E \gg m c^2) \]

\[ P_{\text{cov}} = \frac{2}{3} \frac{e^2}{m^2c^3} \frac{E^4 v^2}{R^2} = \frac{2}{3} \frac{e^2}{m^2c^3} \frac{\epsilon^2 \beta^2 \gamma^2}{R^2} \]

\[ P_{\text{n}} = -e \beta \cdot \gamma = \frac{e R}{2} \frac{d \beta \gamma}{dt} \]

maximum \( E \) when \( P_{\text{rad}} = P_{\text{n}} \)

\[ \frac{2}{3} \frac{e^2}{m^2c^3} \frac{E^4}{R^2} = \frac{e R}{2} \frac{d \beta \gamma}{dt} \]

\[ \frac{E^4}{4} = \frac{3}{4} \frac{m^4c^7}{e} \frac{R^3 d \beta \gamma}{dt} \]

or \( \frac{E^4}{\gamma} = \frac{3}{4} \frac{m^4c^7}{e} \frac{1}{\beta^3} d \beta \gamma \)