J01E.1

This is essentially a boundary value problem, where the voltage of the first lead is v_o, the voltage of the second is 0, and along the edge of the circle, the electric field must be tangential to the circle. This problem is well specified since this final condition amounts to $\partial_r V = 0$, which is a Neumann boundary condition, specifying the normal derivative on the edge.

One way to do this problem is to guess a solution with the proper boundary conditions. A standard textbook problem involves finding the voltage at any point between parallel line charges of opposite sign separated by a distance d. We recall that the solution yields equipotential circles which have the property that they are perpendicular to any circle passing through both line charges. We then adjust λ to enforce the proper potential difference.

In calculating the resistance, we don't care about the absolute potential difference, but only the ratio V/I, so we may as well pick arbitrary λ. The potential due to both rods is then
$V(r_1, r_2) = \frac{\lambda}{2\pi\epsilon_o}\left(\log(r_2/r_b) - \log(r_1/r_a)\right)$ where r_b and r_a specify where reference point of the potential. We may as well pick the zero point to be the origin, where $r_2 = r_1 = d/2$.
$V(r_1, r_2) = \frac{\lambda}{2\pi\epsilon_o}\log\left(\frac{r_2}{r_1}\right)$.

The total voltage between the points is then $\frac{\lambda}{2\pi\epsilon_o}\left(2\log(\frac{d-\delta/2}{\delta/2})\right)$

In the disk, we suppose there is no charge, hence $\sigma\nabla\cdot E = \nabla\cdot J = 0$. This tells us that if we perform a 'surface' integral around any closed curve in the substances, it's total value is going to be zero, according to Gauss' theorem. In particular, let us take any curve which has the edge of the lead which extends into the disk as a boundary. The flux due to J through this surface is the total current flowing in to the disk (with a negative sign). Our curve then follows the boundary of the large disk for a time, and the two ends connect along any arbitrary curve. The flux through this surface is then the total current.

It is most convenient to integrate over the surface of the lead. For $\delta << d$, we can ignore the contribution of the lead which is far away. And take the total current density flux through this surface to be $\sigma t(\pi\delta/2)2\lambda/\delta = \sigma t\lambda\pi$, and

$$R = \frac{V}{I} = \frac{1}{\sigma t\pi^2\epsilon_o} \log(\frac{2d}{\delta} - 1)$$

One thought on "J01E.1"

December 13, 2013 at 6:30 pm

OK, you got it.
