Seth Dorfman Prelims December 26, 2005

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3) a. $n(\varepsilon) = \frac{1}{e^{\beta(\varepsilon-\mu)}-1}$

$N = \frac{V}{(2\pi\hbar)^3}\int_0^\infty n(\varepsilon)\, 4\pi p^2\, dp$ (assume no spin)

$= \frac{V}{(2\pi\hbar^3)}\int_0^\infty \frac{1}{e^{\beta(p^2/2m-\mu)}-1} 4\pi p^2\, dp$

$= \int_0^\infty \frac{V}{2\pi^2\hbar^3} p^2 \frac{1}{e^{\beta(p^2/2m-\mu)}-1} dp$

$\therefore$ the particle distribution in momentum space is

$N(p) = \frac{V}{2\pi^2\hbar^3}\frac{p^2}{e^{\beta(p^2/2m-\mu)}-1}$

b. $\rho \geq \rho_c(T) \Rightarrow \mu = 0$

# of partices in ground state $N_0 = V(\rho-\rho_c)$

$\therefore$ the particle distribution in momentum space is

$N(p) = N_{\text{part a}}(p, \mu=0) + N_0\,\delta(p)$

$N(p) = \frac{V}{2\pi^2\hbar^3}\frac{p^2}{e^{\beta p^2/2m}-1} + V(\rho-\rho_c)\,\delta(p)$

c. $\rho_c(T) = \int_0^\infty \frac{1}{2\pi^2\hbar^3}\frac{p^2}{e^{\beta p^2/2m}-1} dp$ Let $x = \sqrt{\frac{\beta}{2m}}\, p$

$= \frac{1}{2\pi^2\hbar^3}\int_0^\infty \left(\sqrt{\frac{2m}{\beta}}\right)^3 \frac{x^2}{e^{x^2}-1} dx$ $dx = \sqrt{\frac{\beta}{2m}}\, dp$

$\rho_c(T) = \frac{\sqrt{2}}{\pi^2\hbar^3}(mk_BT)^{3/2}\int_0^\infty \frac{x^2}{e^{x^2}-1}dx$

$\rho_c(T) \sim T^{3/2}$

$\Rightarrow \gamma = \frac{3}{2}$

d. Let M = # of particles that escape $M \ll N$

$E_{\text{remaining particles}} = (N-M)e$

$E_{\text{escaping particles}} = MAe$ mean energy of all particles

$E_{tot} = [N + M(A-1)]e = Ne_1$

$\frac{e_1}{e} = 1 + \frac{M}{N}(A-1)$

$\frac{T_i}{T_f} = 1 + \frac{M}{N}(A-1)$

$\left(\frac{T_i}{T_f}\right)^{3/2} \approx 1 + \frac{3}{2}\frac{M}{N}(A-1)$

$\frac{\rho_i}{\rho_f} = \frac{N}{N-M}$ $\frac{\rho_{ci}}{\rho_{cf}} = \left(\frac{T_i}{T_f}\right)^{3/2}$

$\rho_f = \frac{N-M}{N}\rho_i$ $\rho_{cf} = \left(\frac{T_f}{T_i}\right)^{3/2}\rho_{ci}$ $\rho_i = \rho_c(T_i)$

$\rho_f = \left(1-\frac{M}{N}\right)\rho_c(T_i)$ $\rho_{cf} = \left(1-\frac{3}{2}\frac{M}{N}(A-1)\right)\rho_c(T_i)$

$\therefore \rho_f \geq \rho_{cf} \Leftrightarrow 1-\frac{M}{N} \geq \left(1-\frac{3}{2}\frac{M}{N}(A-1)\right)$

$\frac{3}{2}(A-1) \geq 1$

$A \geq \frac{5}{3}$ for the system to remain in the condensed phase