Seth Dorfman Prelims December 26, 2005

January 2000 SM

1) $\mathcal{H}_1 = \frac{p^2}{2m} - g\mu_B \vec{S}\cdot\vec{H}$

$$Z_1 = \int \frac{d^3p\,d^3x}{h^3} e^{-\beta \frac{p^2}{2m}} \cdot \sum_{s=\pm\frac{1}{2}} e^{\beta g \mu_B S H}$$

$$= V\frac{1}{h^3}\left(\pi \frac{2m}{\beta}\right)^{3/2} 2\cosh\left(\frac{\beta g \mu_B H}{2}\right)$$

$$= V\left(\frac{2\pi m k_B T}{4\pi^2\hbar^2}\right)^{3/2} 2\cosh\left(\frac{g\mu_B H}{2k_B T}\right)$$

$$Z_1 = 2V\lambda^{-3}\cosh(x) \qquad \lambda = \sqrt{\frac{2\pi\hbar^2}{mk_BT}} \qquad x = \frac{g\mu_B H}{2k_B T}$$

$$Z = \frac{1}{N!} Z_1^N$$

$$F = -k_B T \ln(Z)$$

$$= -Nk_BT\ln(Z_1) + k_BT\ln(N!)$$

$$= -Nk_BT\ln(2V\lambda^{-3}\cosh(x)) + Nk_BT\ln(N) - Nk_BT$$

$$F = -Nk_BT\left[\ln\left(2\frac{V}{N}\lambda^{-3}\cosh(x)\right) - 1\right]$$

$$\mu = \frac{\partial F}{\partial N}\Big|_{T,V} = -k_BT\left[\ln\left(2\frac{V}{N}\lambda^{-3}\cosh(x)\right) - 1\right] + Nk_BT\cdot\frac{1}{N}$$

$$\mu = -k_BT\ln\left(2\frac{V}{N}\lambda^{-3}\cosh(x)\right)$$

In equilibrium $x_1 = \frac{g\mu_B H}{2k_BT}$, $x_2 = 0$ $T_1 = T_2$, $V_1 = V_2$

Because of the pipe $\mu_1 = \mu_2$

$$\Rightarrow \frac{\cosh(x_1)}{N_1} = \frac{\cosh(x_2)}{N_2}$$

$$\frac{\cosh(x_1)}{\cosh(0)} = \frac{N_1}{N_2}$$

$$\frac{N_1}{N_2} = \cosh(x_1)$$

$$\frac{P_1}{P_2} = \frac{N_1}{N_2} = \cosh(x_1)$$

$$\therefore \frac{P_1}{P_2} = \cosh\left(\frac{g\mu_B H}{2k_BT}\right)$$

where P_1 is the pressure in the cube with the field and P_2 is the pressure in the cube without the field