Two interacting particles have Hamiltonian $H = H_0 + H'$, where:

$$H_0 = -\frac{\hbar^2}{2m}\left(\nabla_1^2 + \nabla_2^2\right) + V\left(\vec{r}_1\right) + V\left(\vec{r}_2\right)$$
$$V\left(\vec{r}\right) = \frac{1}{2}k\left|\vec{r}\right|^2$$
$$H' = \epsilon\left(x_1x_2 + y_1y_2 - 2z_1z_2\right)$$

Let us define ω such that $k = m\omega^2$. The second order energy correction for a non-degenerate state is given by:

$$E_n\left(\epsilon\right) = E_n^0 + \left\langle\varphi_n\right| H' \left|\varphi_n\right\rangle + \sum_{p\neq n}\sum_i \frac{\left|\left\langle\varphi_p^i\right| H' \left|\varphi_n\right\rangle\right|^2}{E_n^0 - E_p^0}$$

We also know that:

$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}\left(a + a^\dagger\right)$$

We can define the eigenstates of the hamiltonian H_0 as $\left|n_{x1}, n_{y1}, n_{z1}, n_{x2}, n_{y2}, n_{z2}\right\rangle$. The creation and annihilation operators act like:

Failed to parse (syntax error): a\left|\varphi_{n}\right\rangle =\sqrt{n}\left|\varphi_{n-1}\right\rangle ;\qquad a^{\dagger}\left|\varphi_{n}\right\rangle =\sqrt{n+1}\left|\varphi_{n+1}\right\rangle

We find that since the operators x_1 and x_2 commute (and similarly for the y 's and z 's) and change the ket by one, the second term (first order correction) is zero. For the second order term, since we are in the ground state, we find nonzero contributions:

$$\left\langle 1,0,0,1,0,0\right| x_1x_2 \left|0,0,0,0,0,0\right\rangle = \frac{\hbar}{2m\omega}$$
$$\left\langle 0,1,0,0,1,0\right| y_1y_2 \left|0,0,0,0,0,0\right\rangle = \frac{\hbar}{2m\omega}$$
$$\left\langle 0,0,1,0,0,1\right| -2z_1z_2 \left|0,0,0,0,0,0\right\rangle = -2\frac{\hbar}{2m\omega}$$

We get the energy of the excited states to be $E_p = \left(\sum_i n_i + \frac{3}{2}\right)\hbar\omega$. The ground state is obviously then at $E_n = \frac{3}{2}\hbar\omega$. Inserting these into our equation for the energy shift, we get:

$$E_n\left(\epsilon\right) = E_n^0 + \frac{\left|\epsilon\frac{\hbar}{2m\omega}\right|^2}{-2\hbar\omega} + \frac{\left|\epsilon\frac{\hbar}{2m\omega}\right|^2}{-2\hbar\omega} + \frac{\left|-2\epsilon\frac{\hbar}{2m\omega}\right|^2}{-2\hbar\omega} = \frac{3}{2}\hbar\omega - 3\frac{\epsilon^2\hbar}{4m^2\omega^3}$$