Seth Dorfman Prelims December 26, 2005

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1) a. In the Born approximation for a central potential $V(r)$, $f(\theta)$ depends on θ only with $\mathcal{K} = 2K\sin(\frac{\theta}{2})$

For precisely backward scattering $\theta = \pi \Rightarrow \mathcal{K} = 2K$

$$\therefore \frac{d\sigma}{d\Omega} = A\frac{1}{(\mathcal{K}/2)^2}e^{-2\lambda\mathcal{K}}$$

$$\frac{d\sigma}{d\Omega} = A\frac{1}{K^2\sin^2(\theta/2)}e^{-4\lambda K\sin(\theta/2)}$$

b. $\frac{d\sigma}{d\Omega} = |f(\theta)|^2$

$$f(\theta) = \frac{m}{\hbar}\hat{V}(\vec{K}-\vec{K}') = \frac{m}{\hbar}\hat{V}(\vec{\mathcal{K}}) \quad \vec{\mathcal{K}} = \vec{K}-\vec{K}'$$

$$\hat{V} = \frac{\hbar}{m}f(\vec{\mathcal{K}})$$

$$V = \frac{\hbar}{m}\int d^3\mathcal{K}\left(\sqrt{A}\frac{1}{\mathcal{K}/2}e^{-\lambda\mathcal{K}}\right)e^{-i\vec{\mathcal{K}}\cdot\vec{r}}$$

$$= \frac{\hbar}{m}\int_0^{2\pi}\int_0^{\pi}\int_0^{\infty}\sqrt{A}\frac{2}{\mathcal{K}}e^{-\lambda\mathcal{K}}e^{-i\mathcal{K}r\cos\theta}\mathcal{K}^2d\mathcal{K}\,d(-\cos\theta)\,d\phi$$

$$= 2\pi\frac{\hbar}{m}\int_0^{\infty}\sqrt{A}\,2\mathcal{K}e^{-\lambda\mathcal{K}}\frac{1}{i\mathcal{K}r}e^{i\mathcal{K}r(-\cos\theta)}\Big|_{\theta=0}^{\theta=\pi}d\mathcal{K}$$

$$= 4\pi\sqrt{A}\frac{\hbar}{m}\int_0^{\infty}\mathcal{K}e^{-\lambda\mathcal{K}}\frac{2}{\mathcal{K}r}\sin(\mathcal{K}r)\,d\mathcal{K}$$

$$= 4\pi\sqrt{A}\frac{\hbar}{mr}\int_0^{\infty}\cancel{\mathcal{K}}e^{-\lambda\mathcal{K}}\frac{2}{\cancel{\mathcal{K}}}\sin(\mathcal{K}r)\,d\mathcal{K}$$

$$= \frac{8\pi\sqrt{A}\hbar}{mr}\,\mathrm{Im}\left[\int_0^{\infty}e^{-\lambda\mathcal{K}}e^{i\mathcal{K}r}d\mathcal{K}\right]$$

$$= \frac{8\pi\sqrt{A}\hbar}{mr}\,\mathrm{Im}\left[\frac{1}{ir-\lambda}e^{\mathcal{K}(ir-\lambda)}\Big|_{\mathcal{K}=0}^{\infty}\right]$$

$$= \frac{8\pi\sqrt{A}\hbar}{mr}\,\mathrm{Im}\left[\frac{1}{\lambda-ir}\right]$$

$$= \frac{8\pi\sqrt{A}\hbar}{mr}\,\mathrm{Im}\left[\frac{\lambda+ir}{\lambda^2+r^2}\right]$$

$$V(r) = \frac{8\pi\sqrt{A}\hbar}{m}\cdot\frac{1}{\lambda^2+r^2}$$