1 January 2000, Quantum Mechanics, Problem 1

1.1 (a)

The formula for the scattering amplitude in the Born approximation is given in Griffiths [1]. For spherically symmetric potentials, there is a simplification that leads to:

$$\frac{d\sigma}{d\Omega} = \frac{m^2}{k^2 sin^2(\theta/2)} \left| \int_0^\infty sin(2ksin(\theta/2)r')V(r')r'\,dr' \right|^2$$

And we know the formula for backwards scattering, so that:

$$\left(\frac{d\sigma}{d\Omega}\right)_{back} = \frac{m^2}{k^2} \left| \int_0^\infty sin(2kr')V(r')r'\,dr' \right|^2 = A\frac{e^{-4\lambda k}}{k^2}$$

Since the angle θ appears in the integrand, however, I do not see how one can extract the differential cross-section at arbitrary angles.

1.2 (b)

Now we want to try to guess the potential that will give us the desired form of the differential cross-section. This also seems impossible to me, unless one already knows what it looks like. See Vasily Pestun's solution for more illumination.

References

[1] D. Griffiths, *Introduction to Quantum Mechanics*.