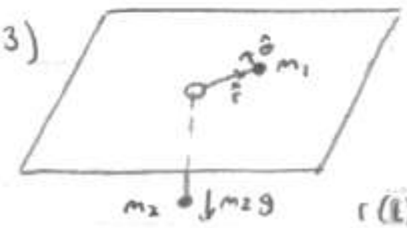
Seth Dorfman Prelims December 23, 2005

January 2000 C.M

3) 

$$\frac{\partial^2}{\partial t^2}(r\hat{r}) = \frac{\partial}{\partial t}(\dot{r}\hat{r} + r\dot{\theta}\hat{\theta})$$

$$\vec{a} = \ddot{r}\hat{r} + \dot{r}\dot{\theta}\hat{\theta} + \dot{r}\dot{\theta}\hat{\theta} + r\ddot{\theta}\hat{\theta} - r\dot{\theta}^2\hat{r}$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + \frac{1}{r}\frac{\partial}{\partial t}(r^2\dot{\theta})\hat{\theta}$$

$$r(t) = r_0 + \varepsilon\cos(w_1 t)$$

No forces on m_1 in $\hat{\theta}$ direction $\Rightarrow \frac{L_z}{m_1} = r^2\dot{\theta} = \text{const}$

$$\frac{L_z}{m_1} = \dot{\theta}\,(r_0^2 + 2r_0\varepsilon\cos(w_1 t) + \underbrace{\varepsilon^2\cos^2(w_1 t)}_{\text{small}})$$

$$\Rightarrow \dot{\theta} = w_0\left(1 - \frac{2\varepsilon}{r_0}\cos(w_1 t)\right)$$

In the $\hat{r}$ direction:

$$(m_1 + m_2)\,a_r = -m_2 g$$

$$m_1(\ddot{r} - r\dot{\theta}^2) + m_2(\ddot{r}) = -m_2 g$$

$$(m_1 + m_2)(-\varepsilon w_1^2\cos(w_1 t)) - m_1 w_0^2\left[(r_0 + \varepsilon\cos(w_1 t))\left(1 - \frac{2\varepsilon}{r_0}\cos(w_1 t)\right)^2\right] = -m_2 g$$

Linearizing

$$(m_1 + m_2)(-\varepsilon w_1^2\cos(w_1 t)) - m_1\left[w_0^2\left[r_0 - r_0\frac{4\varepsilon}{r_0}\cos(w_1 t) + \varepsilon\cos(w_1 t)\right]\right] = -m_2 g$$

$$m_1 w_0^2 r_0 + \varepsilon\cos(w_1 t)\left[(m_1 + m_2)w_1^2 - 3m_1 w_0^2\right] = m_2 g$$

For a circular orbit $\varepsilon = 0$:

$$m_1 w_0^2 r_0 = m_2 g$$

Subtracting this zeroth order equation:

$$\varepsilon\cos(w_1 t)\left[(m_1 + m_2)w_1^2 - 3m_1 w_0^2\right] = 0$$

$$\Rightarrow w_1^2 = \frac{3m_1}{m_1 + m_2}w_0^2$$

For a closed circular orbit with one maximum and one minimum we want $w_1^2 = w_0^2$ or

$$3m_1 = m_1 + m_2$$

$$m_2 = 2m_1$$