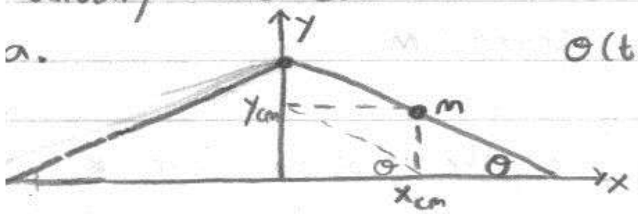
January 2000 CM # 2

a.



$\theta(t=0) \equiv \theta_0$

For one rod $I_{cm} = 2\int_0^{\ell/2} \frac{m}{\ell} r^2\, dr$

$= 2\,\frac{m}{\ell}\,\frac{r^3}{3}\Big|_0^{\ell/2}$

$I_{cm} = \frac{1}{12} m \ell^2$

Due to the symmetry of the problem the x-coordinate of the attachment point remains fixed at $x=0$

$x_{cm} = \frac{\ell}{2}\cos\theta \quad y_{cm} = \frac{\ell}{2}\sin\theta$

$\dot{x}_{cm} = -\frac{\ell}{2}\dot{\theta}\sin\theta \quad \dot{y}_{cm} = \frac{\ell}{2}\dot{\theta}\cos\theta$

$E_{one\ rod} = \frac{1}{2}m(\dot{x}_{cm}^2 + \dot{y}_{cm}^2) + \frac{1}{2}I_{cm}\dot{\theta}^2 + mg\frac{\ell}{2}\sin\theta$

$= \frac{1}{2}m\dot{\theta}^2\frac{\ell^2}{4} + \frac{1}{2}m\dot{\theta}^2\frac{\ell^2}{12} + mg\frac{\ell}{2}\sin\theta$

$= \frac{1}{2}m\dot{\theta}^2\ell^2\left(\frac{3}{12} + \frac{1}{12}\right) + mg\frac{\ell}{2}\sin\theta$

$E_{one\ rod} = \frac{1}{6}m\ell^2\dot{\theta}^2 + mg\frac{\ell}{2}\sin\theta$

By conservation of energy:

$\frac{1}{6}m\ell^2\dot{\theta}^2 + mg\frac{\ell}{2}\sin\theta = mg\frac{\ell}{2}\sin\theta_0$

$\frac{1}{3}\ell\dot{\theta}^2 = g(\sin\theta_0 - \sin\theta)$

$\dot{\theta}^2 = \frac{3g}{\ell}(\sin\theta_0 - \sin\theta) \qquad (1)$

$y_{end} = \ell\sin\theta$

$\dot{y}_{end} = \ell\cos\theta\,\dot{\theta}$

$\dot{y}_{end} = -\sqrt{3g\ell(\sin\theta_0 - \sin\theta)}\,\cos\theta$

Just before the rods hit the table $\theta = 0$

i. $v = \sqrt{3g\ell\sin\theta_0}$

b. $(1) \Rightarrow 2\dot{\theta}\ddot{\theta} = -\frac{3g}{\ell}\cos\theta\,\dot{\theta}$

$\ddot{\theta} = -\frac{3g}{2\ell}\cos\theta$

$\ddot{x}_{cm} = -\frac{\ell}{2}(\dot{\theta}^2\cos\theta + \ddot{\theta}\sin\theta)$

$= -\frac{\ell}{2}\cdot\frac{3g}{\ell}\left((\sin\theta_0 - \sin\theta)\cos\theta - \frac{1}{2}\cos\theta\sin\theta\right)$

$\ddot{x}_{cm} = \frac{3g}{4}(3\sin\theta - 2\sin\theta_0)\cos\theta$

$T = m\ddot{x}_{cm}\big|_{\theta=0}$

$T = \frac{3}{2}mg\sin\theta_0$