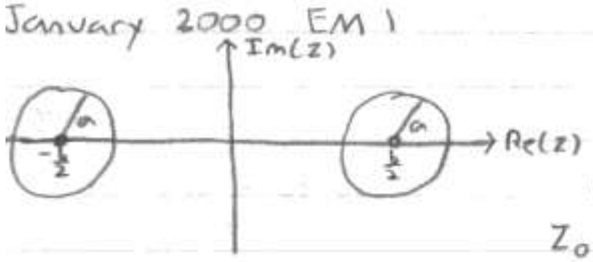
January 2000 EM 1



Find the inverse points common to both circles:

$$(z_0 \pm \tfrac{b}{2})(z_1 \pm \tfrac{b}{2}) = a^2$$

$$z_0 z_1 \pm \tfrac{b}{2}(z_0 + z_1) + (\tfrac{b}{2})^2 = a^2$$

$$\Rightarrow z_1 = -z_0 \quad , \quad z_0^2 = (\tfrac{b}{2})^2 - a^2$$

$$z_0 = \sqrt{(\tfrac{b}{2})^2 - a^2}$$

Let $w = \frac{z - z_0}{z + z_0}$

$$w(z + z_0) = z - z_0$$

$$z(w-1) = z_0(-w-1)$$

$$z = z_0 \frac{1+w}{1-w}$$

The two circles are $|z \pm \frac{b}{2}| = a$

$$|z \pm \tfrac{b}{2}| = \left| z_0 \frac{1+w}{1-w} \pm \frac{b}{2} \right|$$

$$a = \left| \frac{1}{1-w} \left[z_0(1+w) \pm \frac{b}{2}(1-w) \right] \right|$$

$$a e^{i\phi} = \frac{1}{1-w}\left[w(z_0 \mp \tfrac{b}{2}) + z_0 \pm \tfrac{b}{2} \right]$$

$$a e^{i\phi} - w a e^{i\phi} = w(z_0 \mp \tfrac{b}{2}) + z_0 \pm \tfrac{b}{2}$$

$$w(z_0 + a e^{i\phi} \mp \tfrac{b}{2}) = -z_0 + a e^{i\phi} \mp \tfrac{b}{2}$$

$$w(z_0 + a e^{i\phi} \mp \tfrac{b}{2})(z_0 + a e^{-i\phi} \mp \tfrac{b}{2}) = (-z_0 + a e^{i\phi} \mp \tfrac{b}{2})(z_0 + a e^{-i\phi} \mp \tfrac{b}{2})$$

$$w\left[z_0^2 + a^2 + (\tfrac{b}{2})^2 + (z_0 \mp \tfrac{b}{2})(a e^{i\phi} + a e^{-i\phi}) \mp z_0 b \right]$$

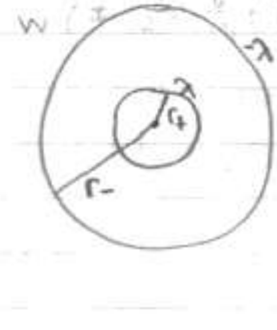
$$= -z_0^2 + a^2 + (\tfrac{b}{2})^2 + z_0(a e^{i\phi} - a e^{-i\phi}) \mp \tfrac{b}{2}(a e^{i\phi} + a e^{-i\phi})$$

$$w\left[2(\tfrac{b}{2})^2 + (z_0 \mp \tfrac{b}{2}) 2a\cos\phi \mp z_0 b \right] = 2a^2 \mp ab\cos\phi + 2a z_0 i \sin\phi$$

$$w(z_0 \mp \tfrac{b}{2})\left[\mp b + 2a\cos\phi \right] = a(2a \mp b\cos\phi + 2i z_0 \sin\phi)$$

$$w(\phi = \tfrac{\pi}{2}) = \frac{2i z_0}{\frac{b}{2} \mp z_0} \frac{a}{b} = i r_\mp$$

(real axis maps to real axis thus this gives the radius)



$$2\pi r E \ell = 4\pi \lambda \ell$$

$$\vec{E} = \frac{2\lambda}{r}\hat{r}$$

$$V = \int_{r_+}^{r_-} E(r)\, dr$$

$$V = \int_{r_+}^{r_-} \frac{2\lambda}{r}\, dr$$

$$V = 2\lambda \ln\left(\frac{r_-}{r_+}\right) = 2\lambda \ln\left(\frac{b/2 + z_0}{b/2 - z_0}\right)$$

$$V = 2\lambda \ln\left(\frac{(b/2 + z_0)^2}{(b/2)^2 - z_0^2}\right)$$

$$V = 4\lambda \ln\left(\frac{b/2 + z_0}{a}\right)$$

$$CV = \lambda \Rightarrow C = \frac{1}{4\ln\left(\frac{b/2 + z_0}{a}\right)}$$

$$C = \frac{1}{4\ln\left(\frac{b/2 + \sqrt{(b/2)^2 - a^2}}{a}\right)}$$